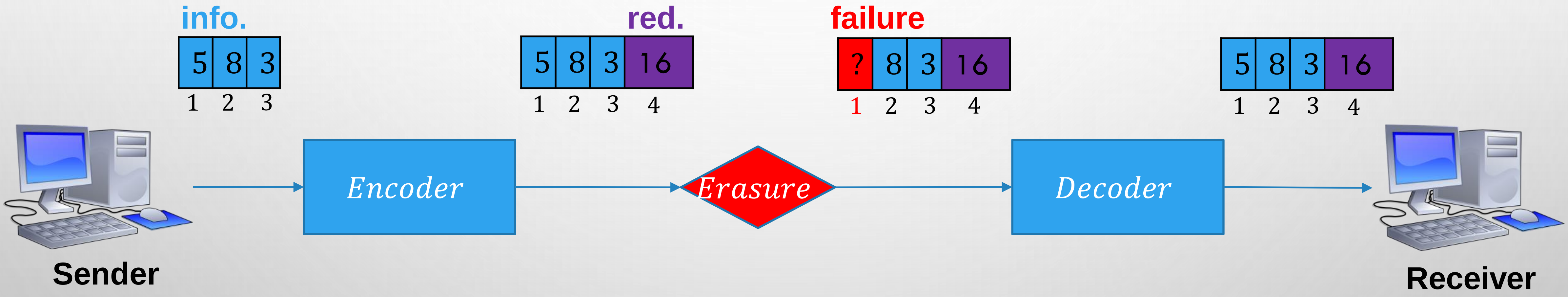
Codes for Erasures over Directed Graphs

Lev Yohananov and Eitan Yaakobi

Technion – Israel Institute of Technology

Classical Erasure Channel

Vectors



info.

5	8	3
1	2	3

red.

5	8	3	16
1	2	3	4

failure

?	8	3	16
1	2	3	4

5	8	3	16
1	2	3	4

Sender

Encoder

Erasure

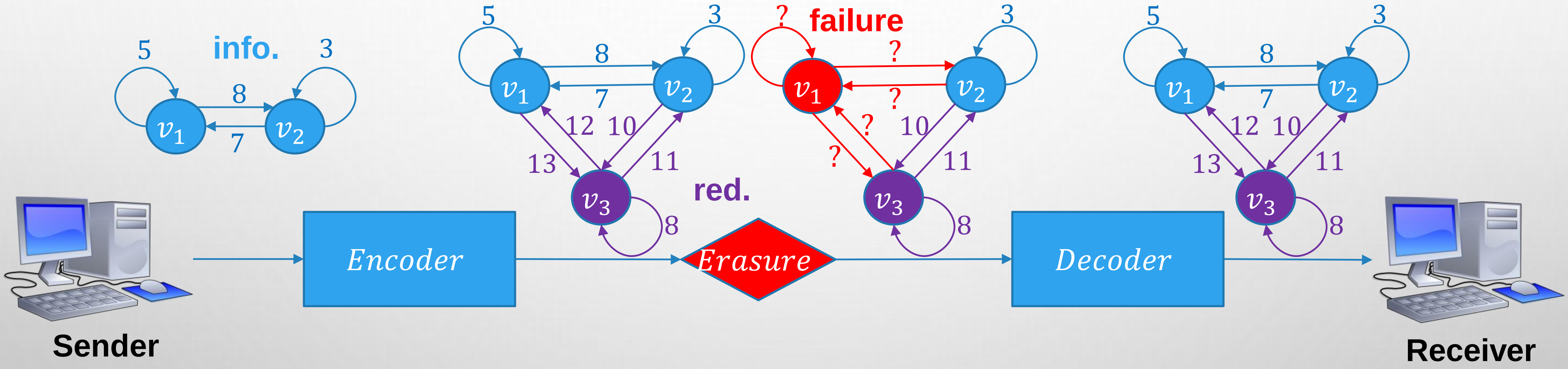
Decoder

Receiver

The indices of the erased values are known

Graph Erasure Channel

Labeled complete directed graphs with self loops



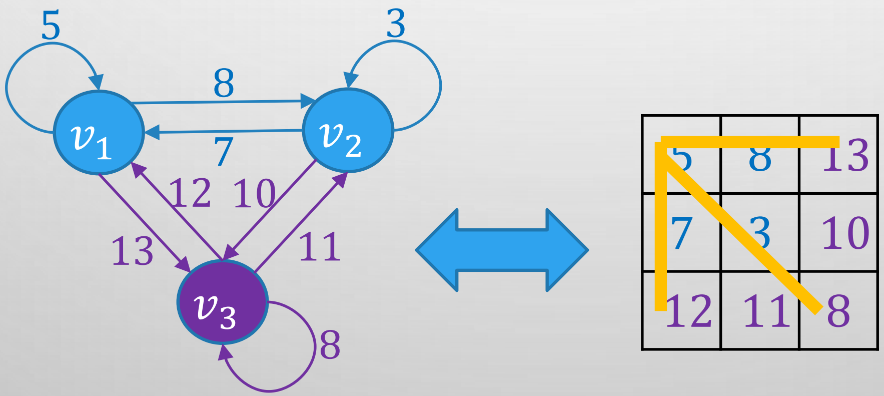
The indices of the erased nodes are known

Motivation

- **Systems where information is represented by a graph**
  - *Neural networks, associative memories, distributed systems*
- **Codes over graphs store information on edges**
  - *Tolerating node failures*

Code over Directed Graphs

- A ***graph code*** of length n over Σ is a set of ***M*** **complete directed graphs with self loops** $C_G = \{G_i = (V_i, L_i)\}_{i=1}^{M}$ **over** Σ **where** $|V_i| = n$
  - L_i is a label function $L_i : V_i \times V_i \to \Sigma$
- **Every graph** G_i **can be represented by an adjacency matrix** A_{G_i}

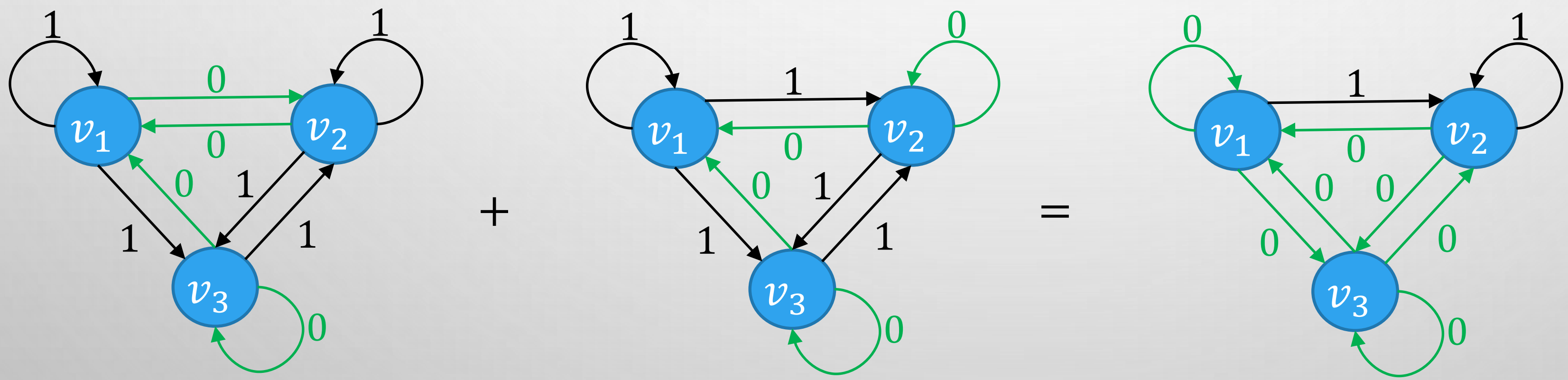


5	8	13
7	3	10
12	11	8

Linear Code over Directed Graphs

- **The operators $+$ and ● for codes over graphs over Σ**
    - *For example* $\Sigma = \mathbb{F}_2$

$$\begin{array}{|c|c|c|}\hline 1 & 0 & 1 \\ \hline 0 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$



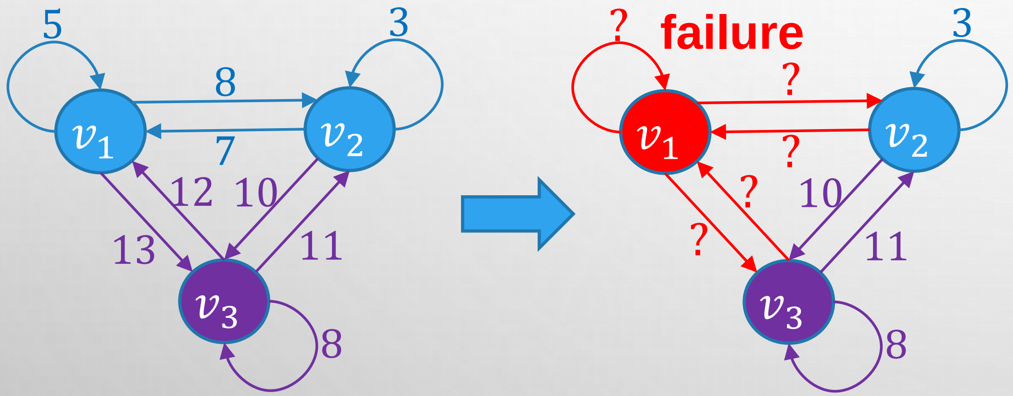
- $\mathcal{G} - [n, k_G]_\Sigma$ **- linear code over directed graphs over Σ, where k_G is the dimension of the code, $k_G = \log M$ where M is a number of graphs in the code**

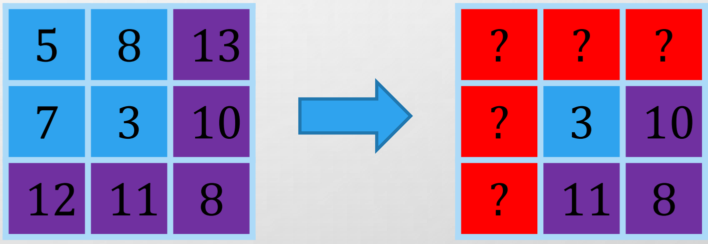
Systematic Code over Directed Graphs

- **A systematic code over graphs has k nodes storing the information symbols on their edges**
  - We denote such a code by $\mathcal{SG}-[n,k]_\Sigma$
    - Binary case $\mathcal{SG}-[n,k]$
    - $k_G = k^2$
- **Summary:** Information nodes - k, information edges - k_G, redundancy nodes - r, redundancy edges - r_G

Node Failure

- **A node failure is the event, where all edges in the neighborhood of the failed node are erased**
  - The failed nodes are known





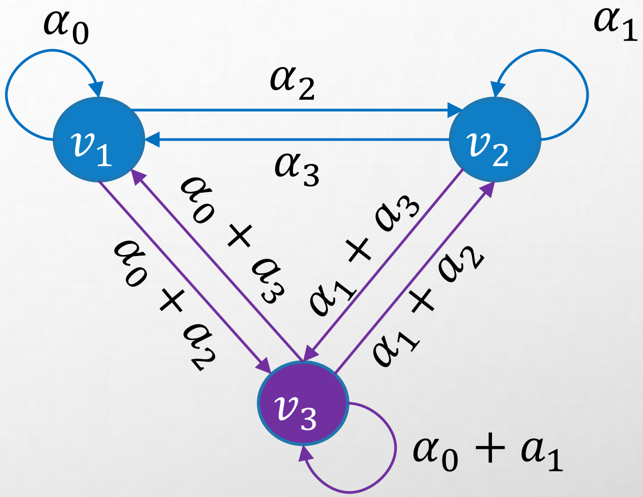
5	8	13
7	3	10
12	11	8

?	?	?
?	3	10
?	11	8

- **A code over graphs is called a ρ-node-erasure-correcting code, if it can correct the failure of any ρ nodes**

Example: $\mathcal{SG}-[n=3,k=2],\boldsymbol{M}=\mathbf{16}$

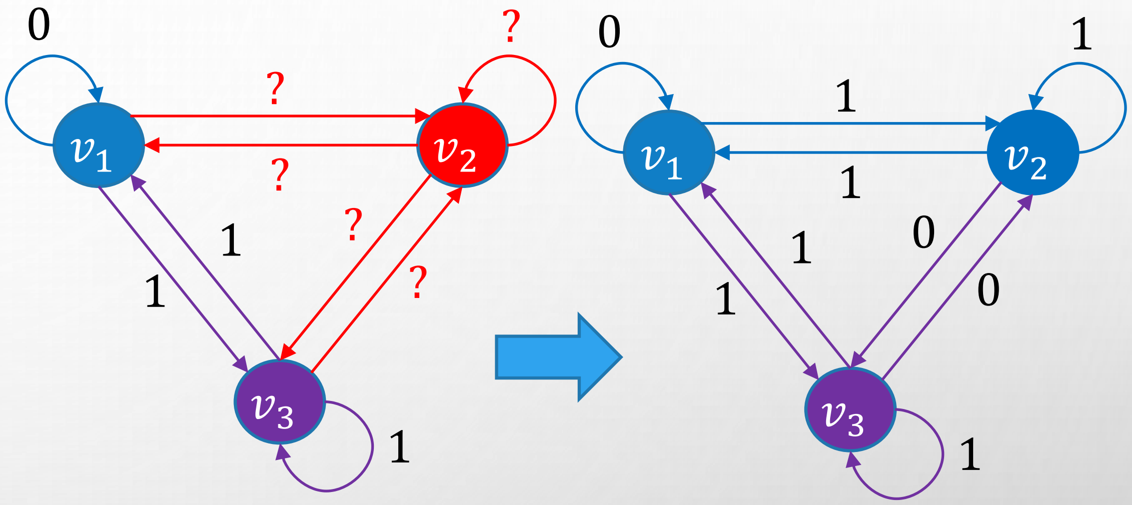
- **Information nodes:** v_1, v_2 $k=2$
- **Information edges:** $\alpha_0, \alpha_1, \alpha_2, \alpha_3 \in \{0,1\}$ $k_G = 4$
- **Redundancy node:** v_3 $r=1$
- **Redundancy edges** are purple $r_G = 5$

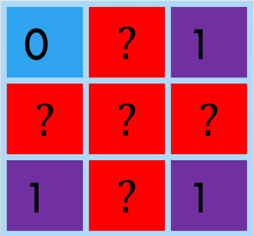


α_0	α_2	$\alpha_0+\alpha_2$
α_3	α_1	$\alpha_1+\alpha_3$
$\alpha_0+\alpha_3$	$\alpha_1+\alpha_2$	$\alpha_0+\alpha_1$

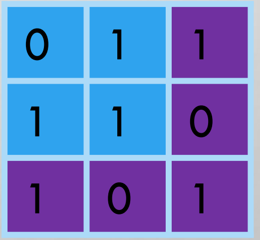
Example: $\mathcal{SG}-[n=3,k=2], \boldsymbol{M=16}$

- **Information nodes:** v_1, v_2 $k=2$
- **Information edges:** $\alpha_0, \alpha_1, \alpha_2, \alpha_3 \in \{0,1\}$ $k_G=4$
- **Redundancy node:** v_3 $r=1$
- **Redundancy edges are purple** $r_G=5$





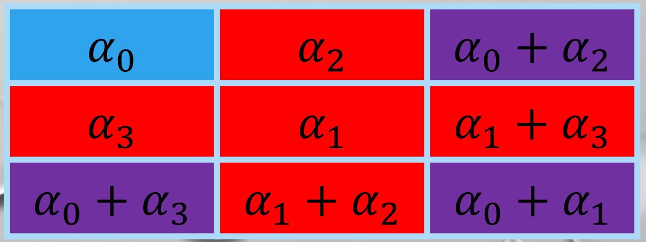
0	?	1
?	?	?
1	?	1



0	1	1
1	1	0
1	0	1

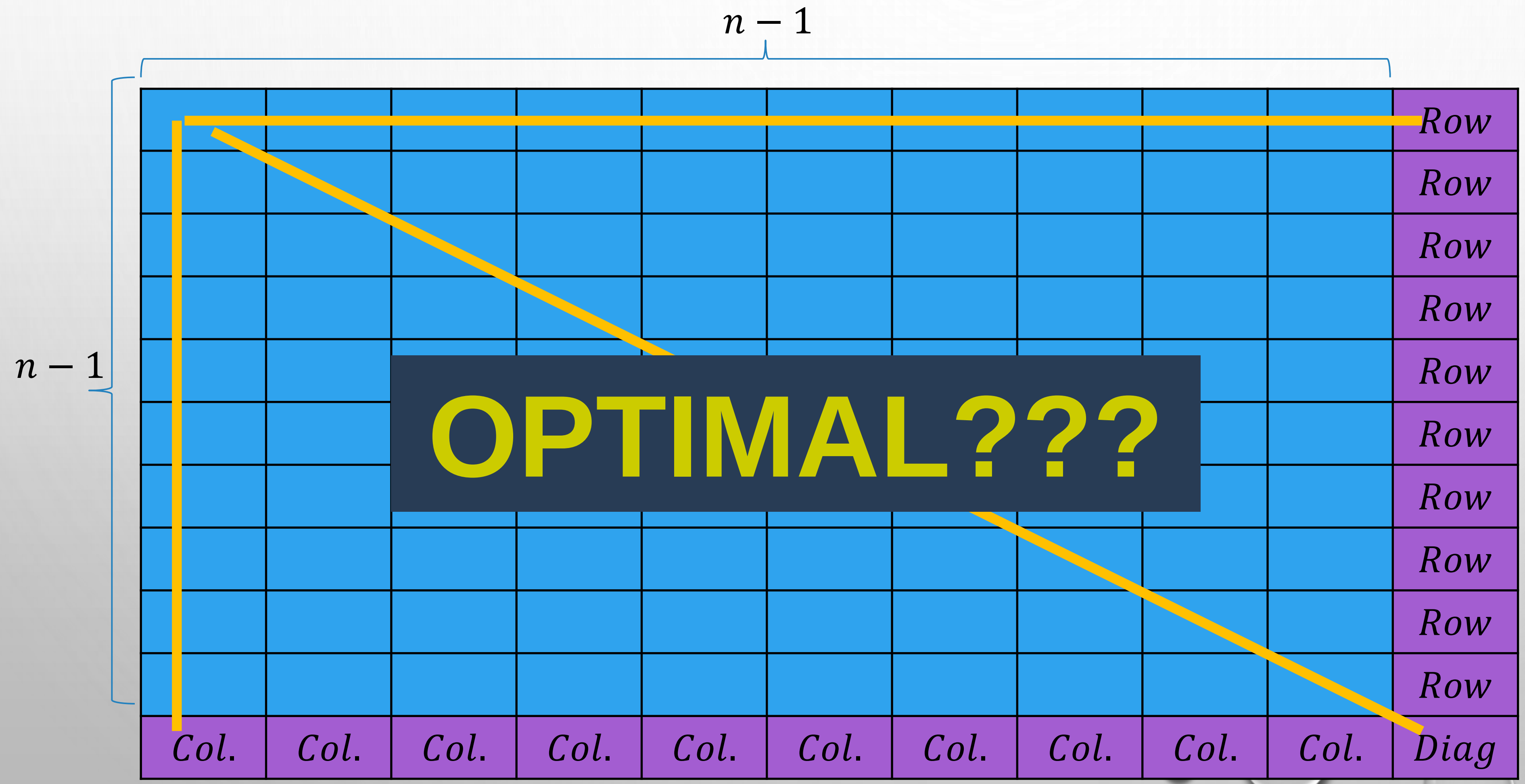
Tolerating every single node failure

- **Rows and columns are simple parity**
- **Main diagonal is a simple parity**
- **Works for all** n



α_0	α_2	$\alpha_0+\alpha_2$
α_3	α_1	$\alpha_1+\alpha_3$
$\alpha_0+\alpha_3$	$\alpha_1+\alpha_2$	$\alpha_0+\alpha_1$

Example: $\mathcal{SG}-[n,n-1], \boldsymbol{M}=\boldsymbol{2^{k_G}}$



$n-1$

$n-1$

Row
Row
Row
Row
Row
Row
Row
Row
Row
Row

Col. Col. Col. Col. Col. Col. Col. Col. Col. Col. Diag

OPTIMAL???

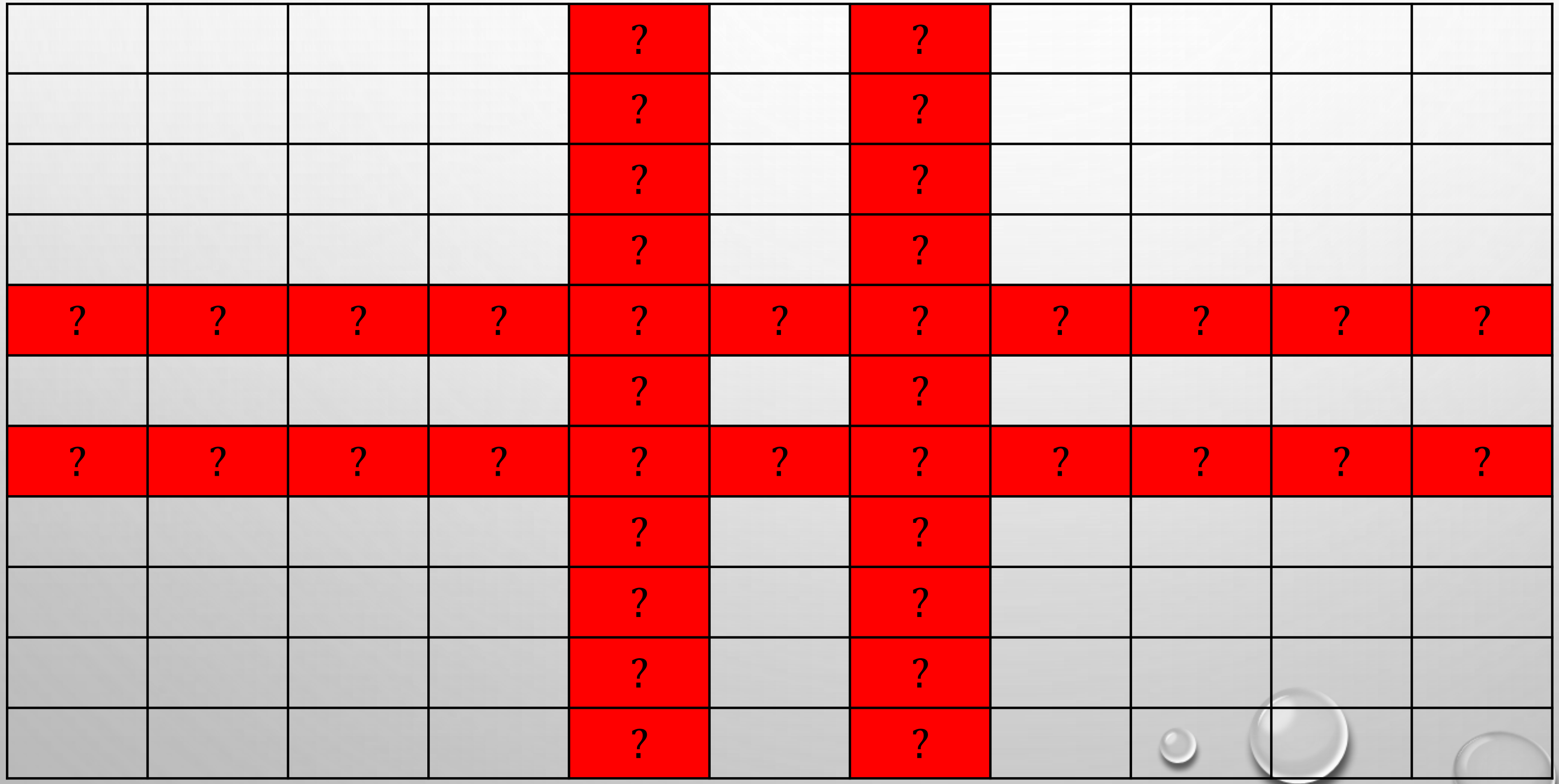
A Pattern of Erasure in Matrix Representation

- A complete graph with self loops
- Each edge has a label

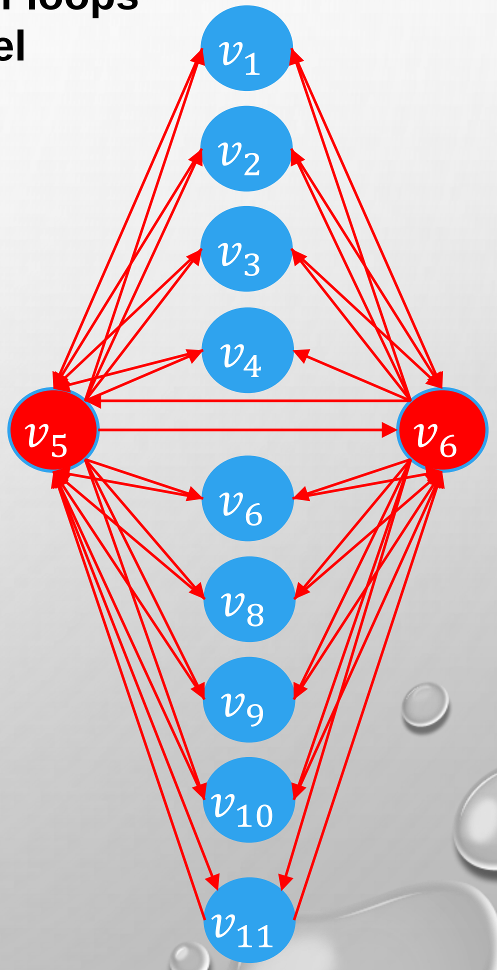
A Pattern of Erasure in Matrix Representation

- The nodes v_5 and v_7 are erased

- A complete graph with self loops
- Each edge have some label



				?		?				
				?		?				
				?		?				
				?		?				
?	?	?	?	?	?	?	?	?	?	?
				?		?				
?	?	?	?	?	?	?	?	?	?	?
				?		?				
				?		?				
				?		?				
				?		?				

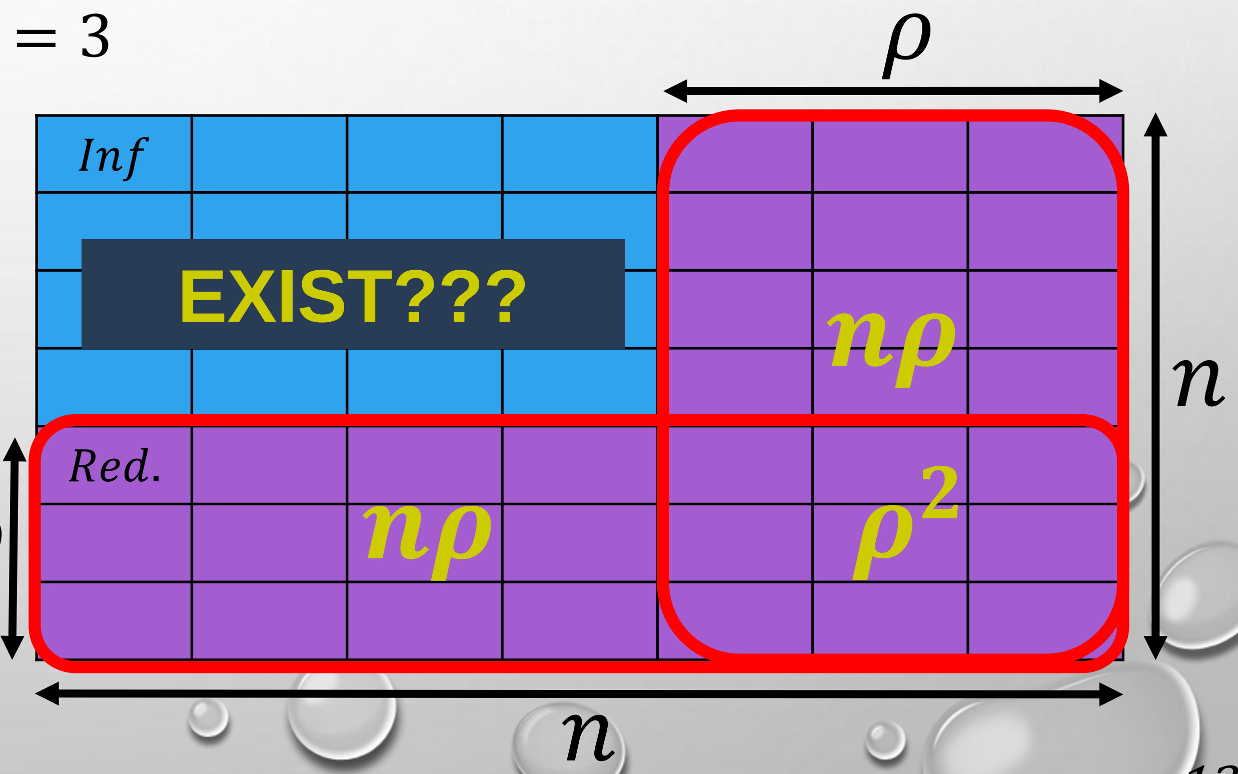


Redundancy Bound

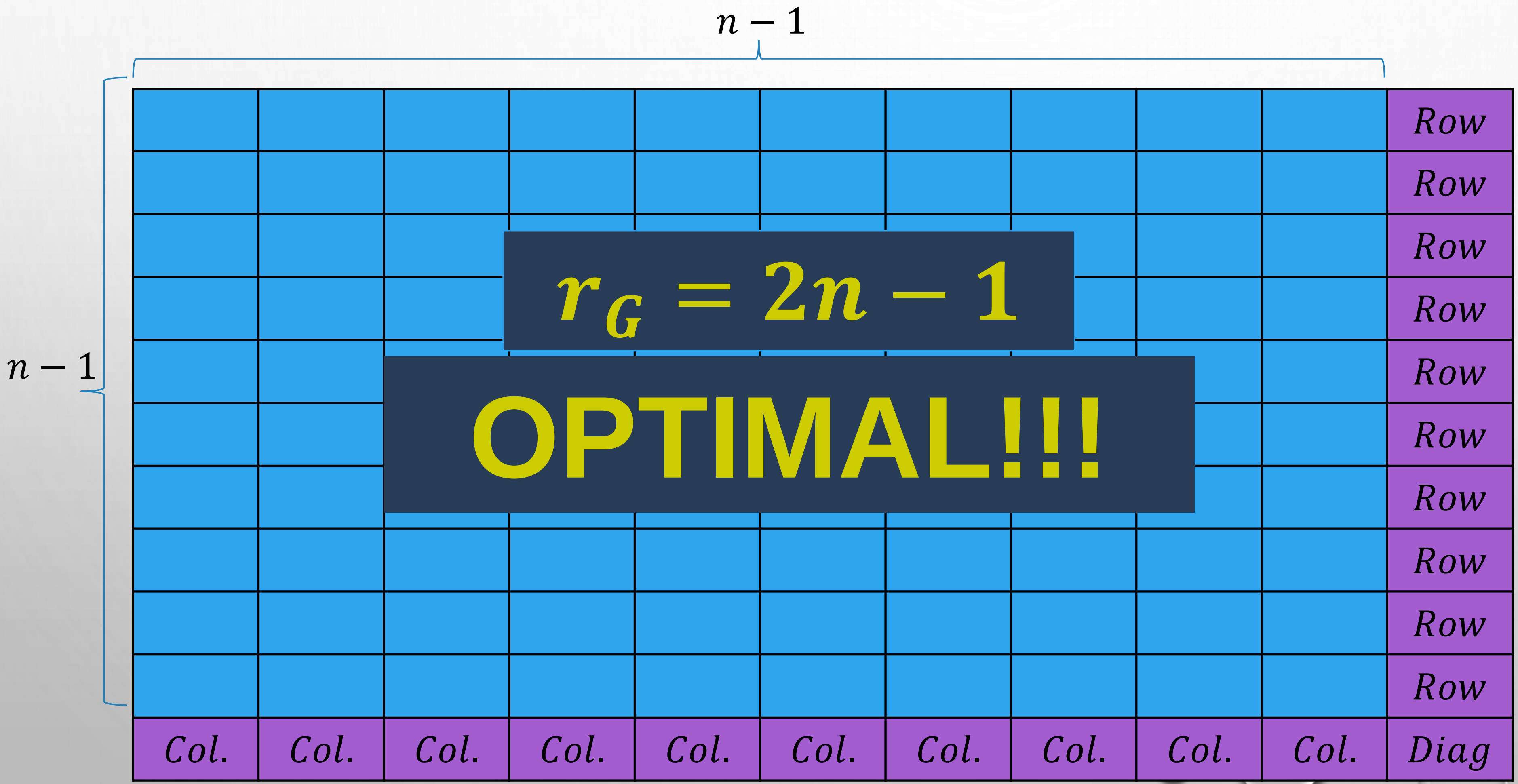
- **Singleton bound:** **For a given ρ, the number of redundancy edges r_G satisfies $r_G \geq 2n\rho - \rho^2$**
  - *For systematic codes $r \geq \rho$*
- **For example: $\mathcal{SG} - [n = 7, k = 4], \rho = 3$**

$$r_G = 2n\rho - \rho^2 = 2 \cdot 7 \cdot 3 - 3^2 = 33$$

ρ – **number of failed nodes**
r – **number of redundancy nodes**
r_G – **number of redundancy edges**



Example: $\mathcal{SG}-[n,n-1], \boldsymbol{M}=\mathbf{2}^{\boldsymbol{k_G}}$

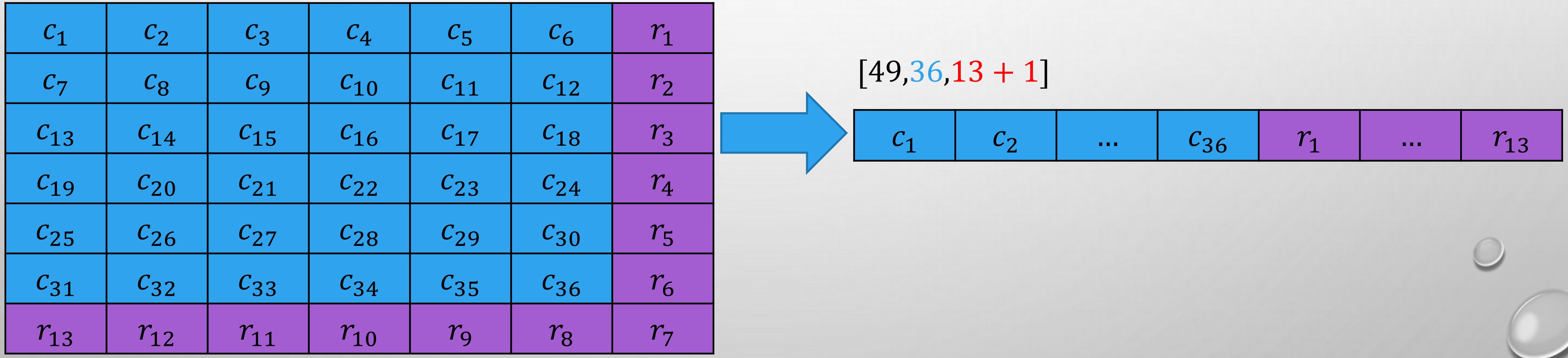


$$r_G = 2n-1$$

OPTIMAL!!!

Trivial Construction

- **The construction of optimal graph codes can be accomplished by** $[n^2, (n-\rho)^2, 2n\rho-\rho^2+1]$ **MDS codes over a field of size** $\boldsymbol{q \geq n^2-1}$
  - *For example*: $\mathcal{SG}-[n=7, k=6]_q, \rho=1, r_G=13$



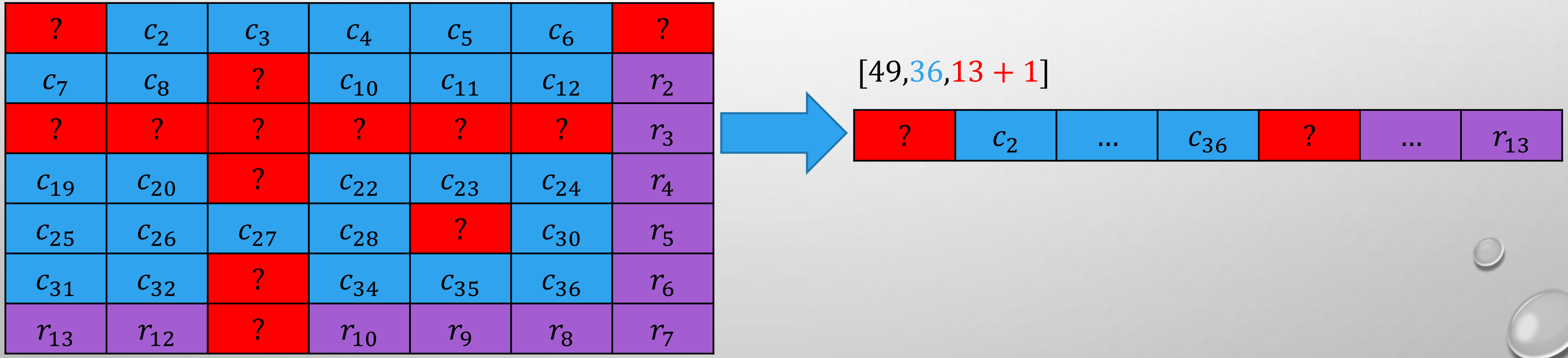
c_1	c_2	c_3	c_4	c_5	c_6	r_1
c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	r_2
c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	r_3
c_{19}	c_{20}	c_{21}	c_{22}	c_{23}	c_{24}	r_4
c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}	r_5
c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	r_6
r_{13}	r_{12}	r_{11}	r_{10}	r_9	r_8	r_7

$[49, 36, 13+1]$

c_1	c_2	…	c_{36}	r_1	…	r_{13}

Trivial Construction

- **The construction of optimal graph codes can be accomplished by $[n^2,(n-\rho)^2,2n\rho-\rho^2+1]$ MDS codes over a field of size $\boldsymbol{q \geq n^2-1}$**
  - *For example*: $\mathcal{SG}-[n=7,k=6]_q,\rho=1,r_G=13$



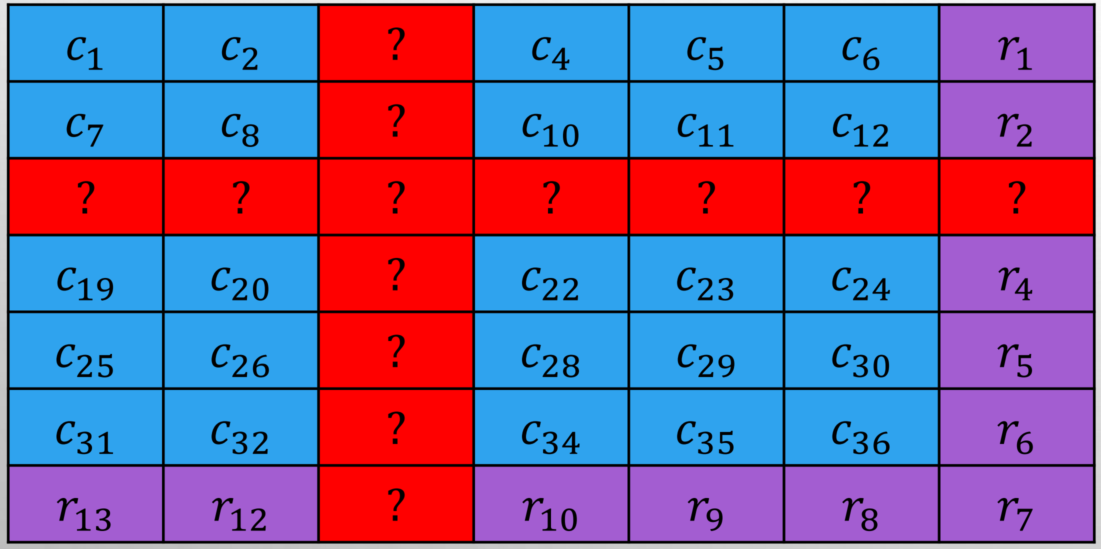
?	c_2	c_3	c_4	c_5	c_6	?
c_7	c_8	?	c_{10}	c_{11}	c_{12}	r_2
?	?	?	?	?	?	r_3
c_{19}	c_{20}	?	c_{22}	c_{23}	c_{24}	r_4
c_{25}	c_{26}	c_{27}	c_{28}	?	c_{30}	r_5
c_{31}	c_{32}	?	c_{34}	c_{35}	c_{36}	r_6
r_{13}	r_{12}	?	r_{10}	r_9	r_8	r_7

$[49,36,13+1]$

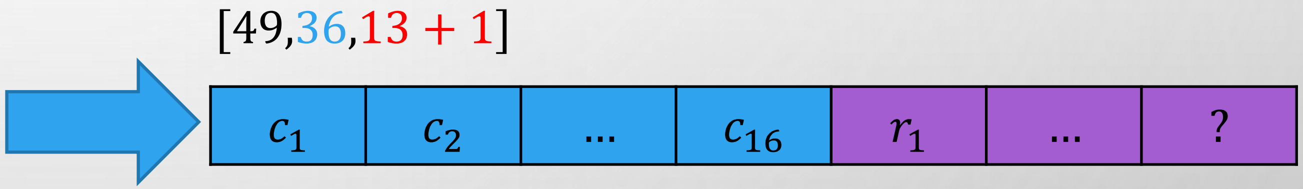
?	c_2	…	c_{36}	?	…	r_{13}

Trivial Construction

- **The construction of optimal graph codes can be accomplished by $[n^2, (n-\rho)^2, 2n\rho - \rho^2 + 1]$ MDS codes over a field of size $q \geq n^2 - 1$**
  - *For example*: $\mathcal{SG} - [n = 7, k = 6]_q, \rho = 1, r_G = 13$



c_1	c_2	?	c_4	c_5	c_6	r_1
c_7	c_8	?	c_{10}	c_{11}	c_{12}	r_2
?	?	?	?	?	?	?
c_{19}	c_{20}	?	c_{22}	c_{23}	c_{24}	r_4
c_{25}	c_{26}	?	c_{28}	c_{29}	c_{30}	r_5
c_{31}	c_{32}	?	c_{34}	c_{35}	c_{36}	r_6
r_{13}	r_{12}	?	r_{10}	r_9	r_8	r_7



$[49, 36, 13 + 1]$

c_1	c_2	…	c_{16}	r_1	…	?

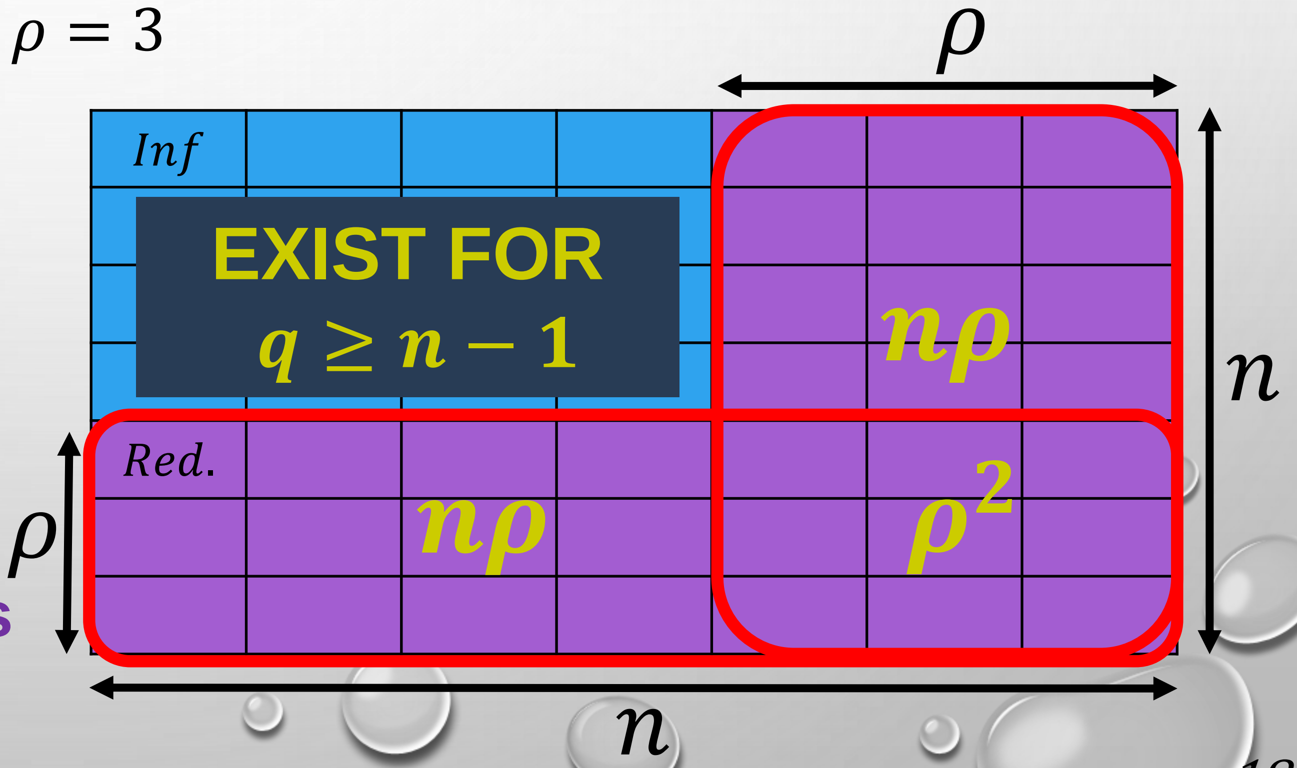
- **Smaller fields: binary or linear**
- **Optimal redundancy**

Redundancy Bound

- **Singleton bound:** **For a given ρ, the number of redundancy edges r_G satisfies $r_G \geq 2n\rho - \rho^2$**
  - *For systematic codes $r \geq \rho$*
- **For example: $\mathcal{SG} - [n = 7, k = 4], \rho = 3$**

$$r_G = 2n\rho - \rho^2 = 2 \cdot 7 \cdot 3 - 3^2 = 33$$

ρ – **number of failed nodes**
r – **number of redundancy nodes**
r_G – **number of redundancy edges**

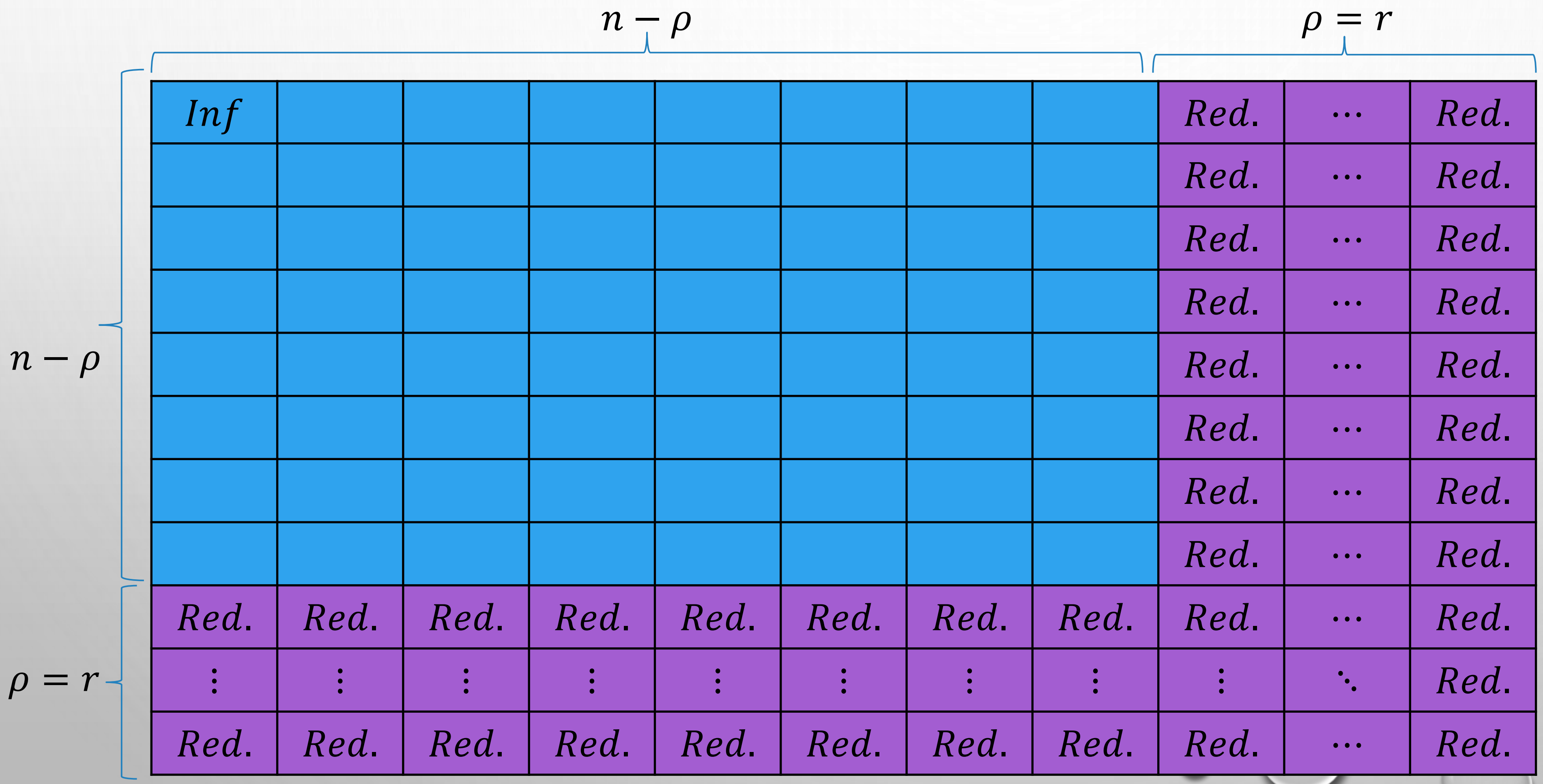


EXIST FOR
$q \geq n - 1$

Optimal Linear ρ-Node-Erasure-Correcting Codes

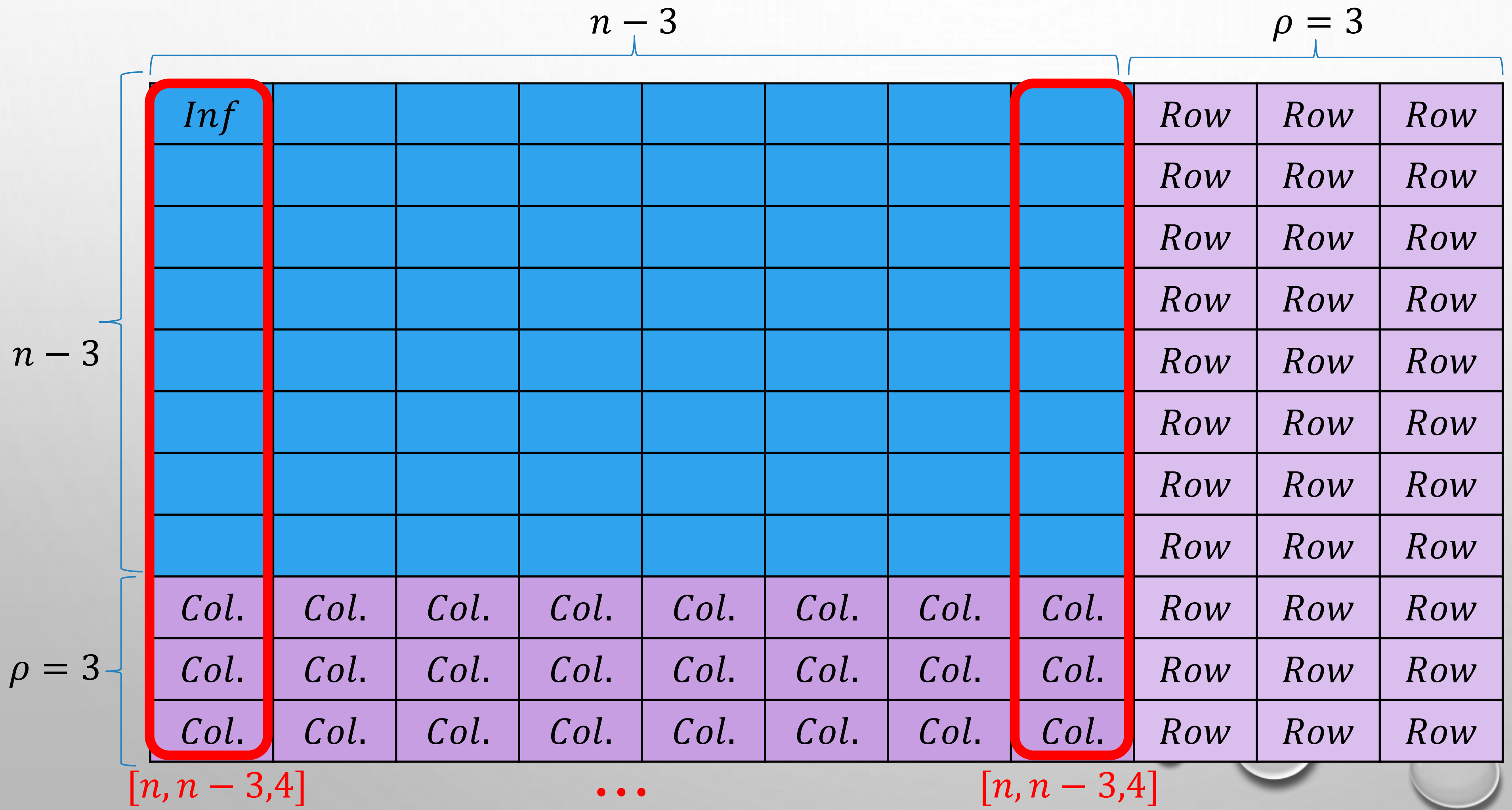
$\mathcal{SG}-[n,n-\rho]_q$ $\boldsymbol{q}$ is a prime power greater than $\boldsymbol{n-1}$

Optimal Linear ρ-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-\rho]_q$



Inf								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
								$Red.$	$\cdots$	$Red.$
$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$\cdots$	$Red.$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$Red.$
$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$Red.$	$\cdots$	$Red.$

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



$n-3$

$\rho=3$

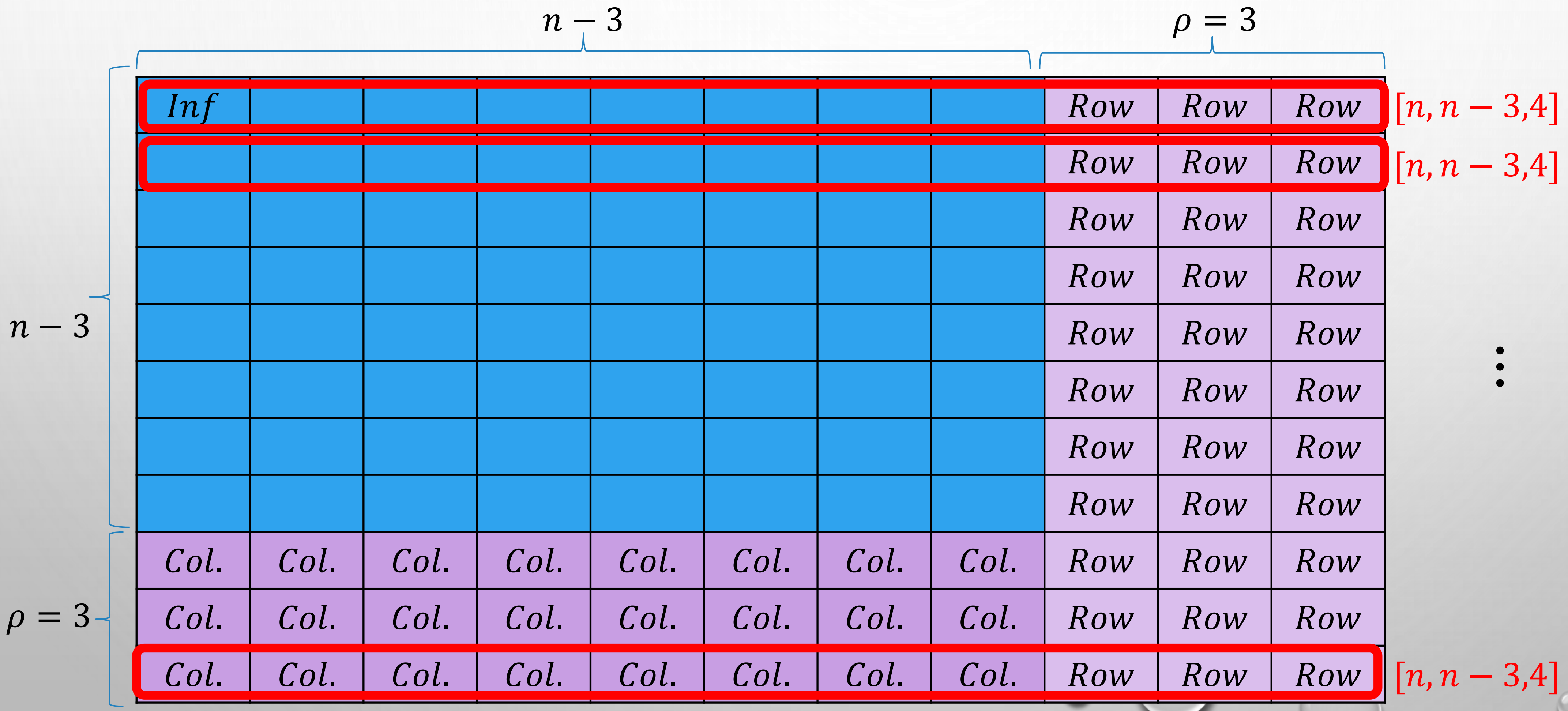
Inf								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row

$n-3$

$\rho=3$

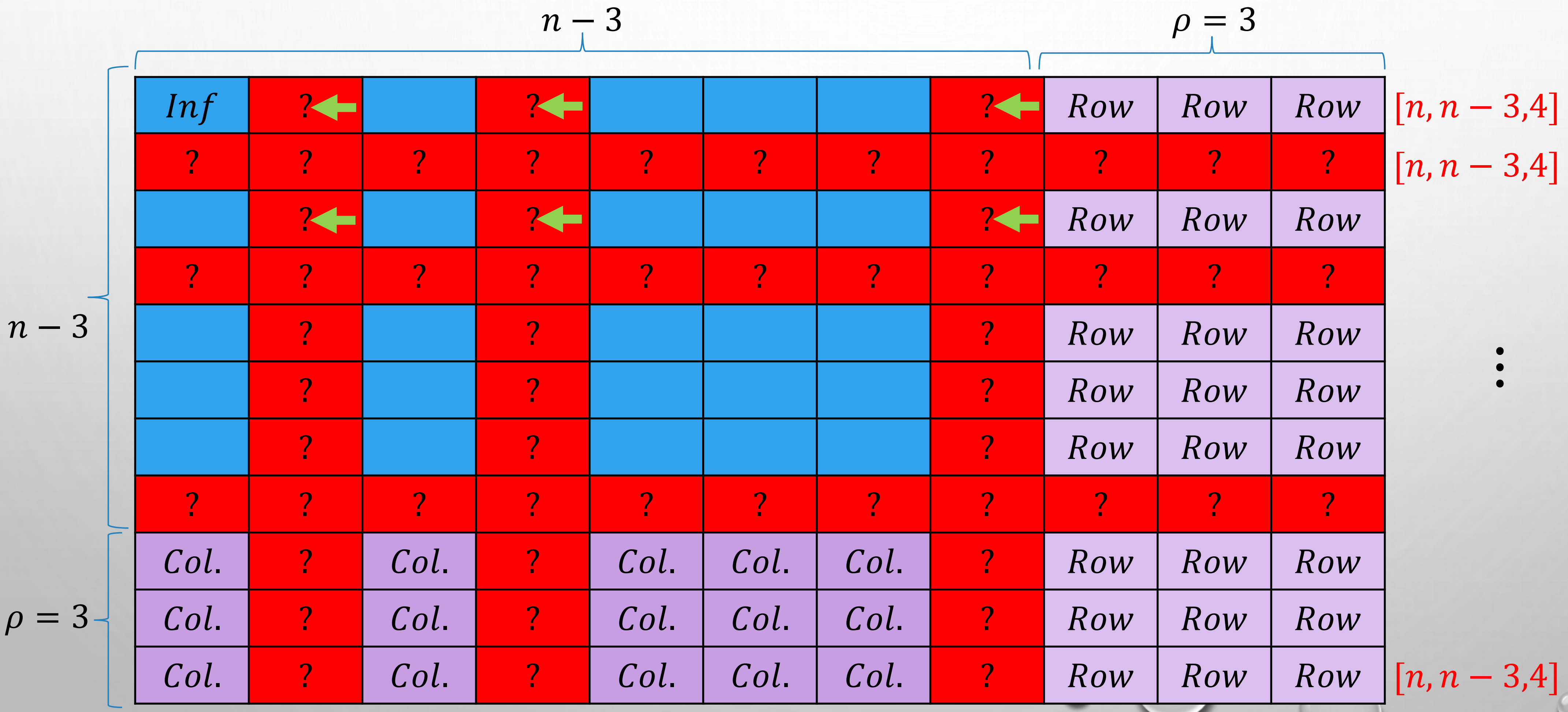
$[n,n-3,4]$ $\cdots$ $[n,n-3,4]$

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



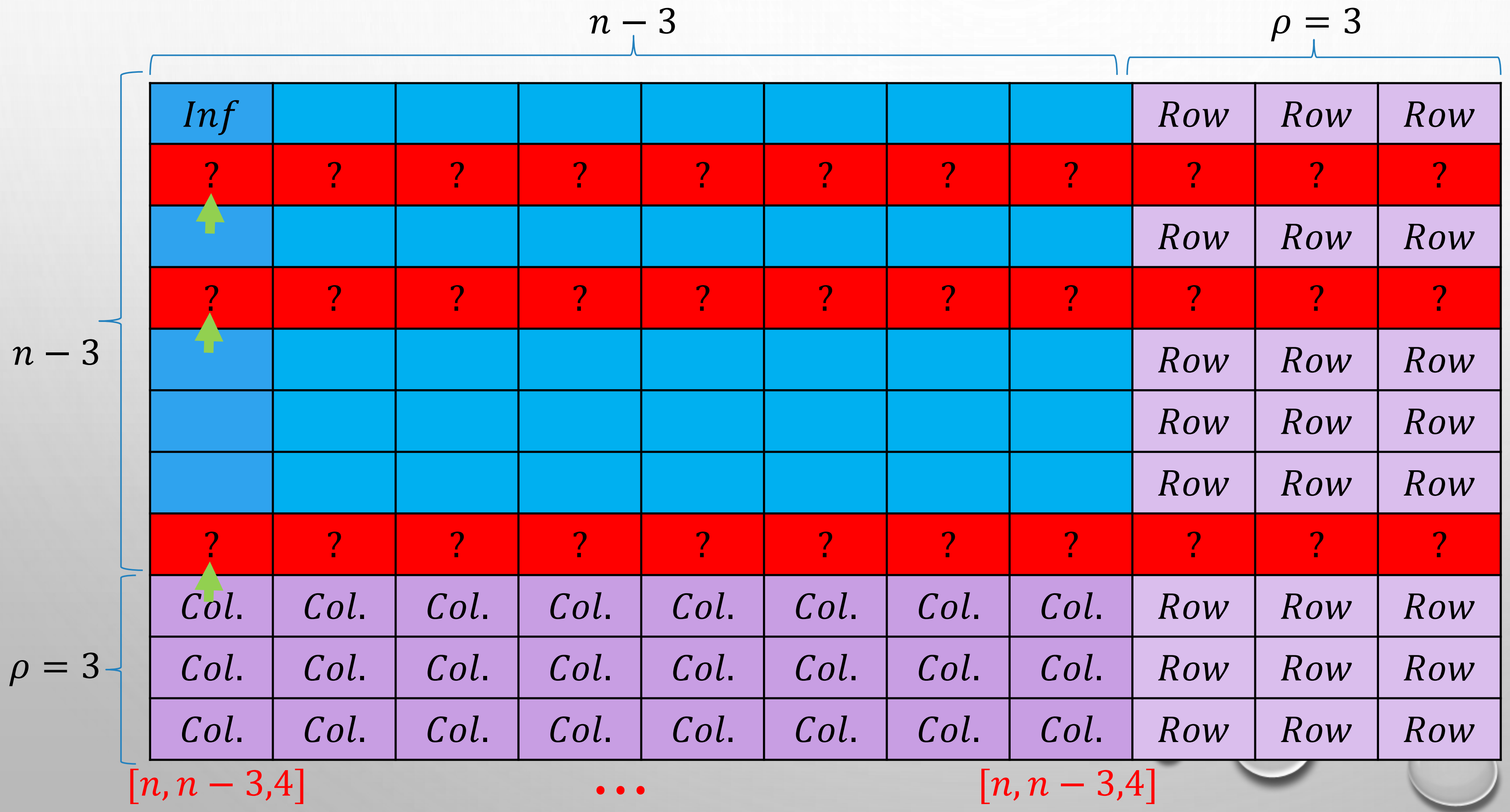
	$n-3$								$\rho=3$			
$n-3$	Inf								Row	Row	Row	$[n,n-3,4]$
									Row	Row	Row	$[n,n-3,4]$
									Row	Row	Row	
									Row	Row	Row	
									Row	Row	Row	⋮
									Row	Row	Row	
									Row	Row	Row	
									Row	Row	Row	
$\rho=3$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	
	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	
	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	$[n,n-3,4]$

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



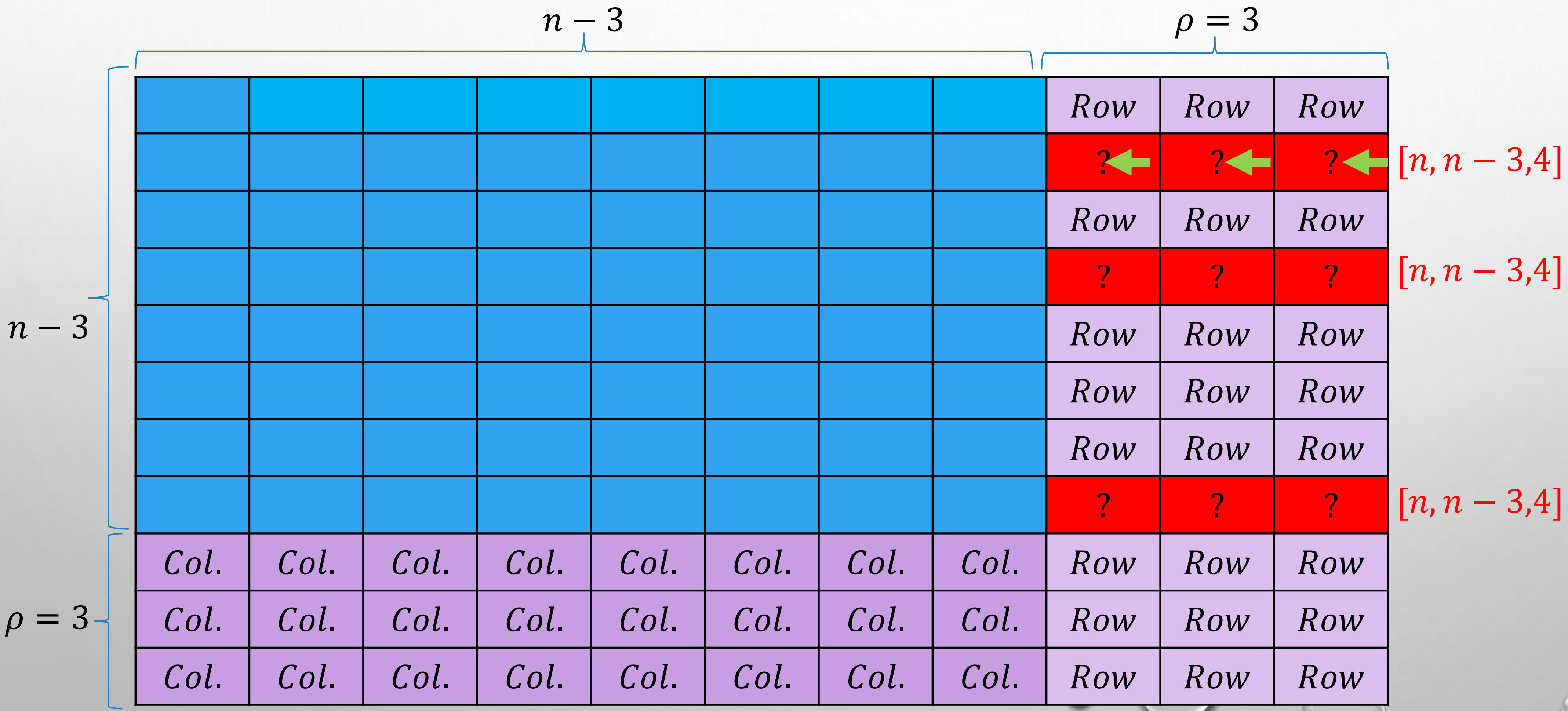
	$n-3$								$\rho=3$			
$n-3$	Inf	?←	🟦	?←	🟦	🟦	🟦	?←	Row	Row	Row	$[n,n-3,4]$
	?	?	?	?	?	?	?	?	?	?	?	$[n,n-3,4]$
	🟦	?←	🟦	?←	🟦	🟦	🟦	?←	Row	Row	Row	
	?	?	?	?	?	?	?	?	?	?	?	
	🟦	?	🟦	?	🟦	🟦	🟦	?	Row	Row	Row	⋮
	🟦	?	🟦	?	🟦	🟦	🟦	?	Row	Row	Row	
	🟦	?	🟦	?	🟦	🟦	🟦	?	Row	Row	Row	
	?	?	?	?	?	?	?	?	?	?	?	
$\rho=3$	$Col.$	?	$Col.$	?	$Col.$	$Col.$	$Col.$	?	Row	Row	Row	
	$Col.$	?	$Col.$	?	$Col.$	$Col.$	$Col.$	?	Row	Row	Row	
	$Col.$	?	$Col.$	?	$Col.$	$Col.$	$Col.$	?	Row	Row	Row	$[n,n-3,4]$

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



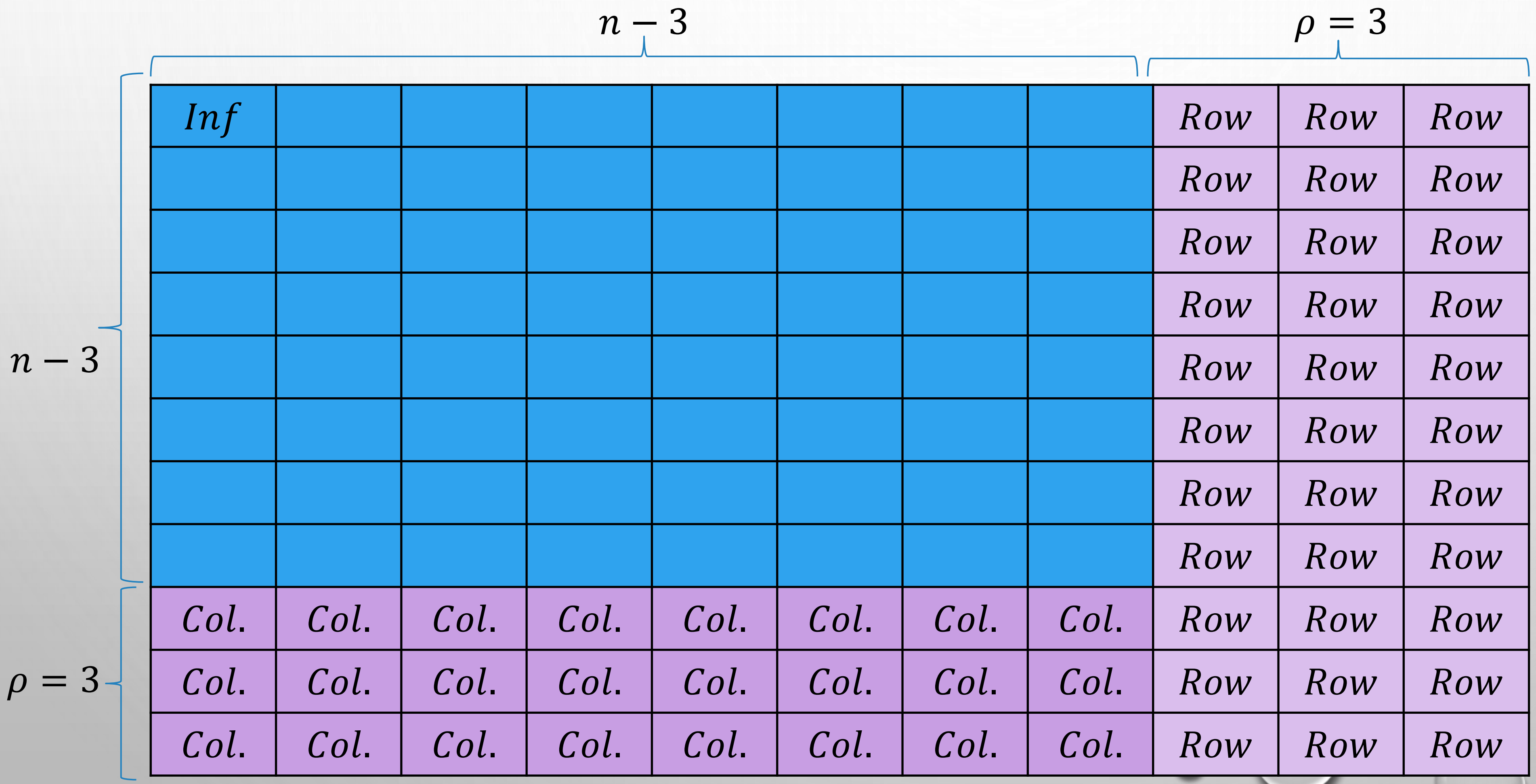
Inf								Row	Row	Row
?	?	?	?	?	?	?	?	?	?	?
								Row	Row	Row
?	?	?	?	?	?	?	?	?	?	?
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
?	?	?	?	?	?	?	?	?	?	?
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



	$n-3$								$\rho=3$			
$n-3$									Row	Row	Row	
									? ←	? ←	? ←	$[n,n-3,4]$
									Row	Row	Row	
									?	?	?	$[n,n-3,4]$
									Row	Row	Row	
									Row	Row	Row	
									Row	Row	Row	
									?	?	?	$[n,n-3,4]$
$\rho=3$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	
	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	
	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row	

Optimal Linear 3-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-3]_q$



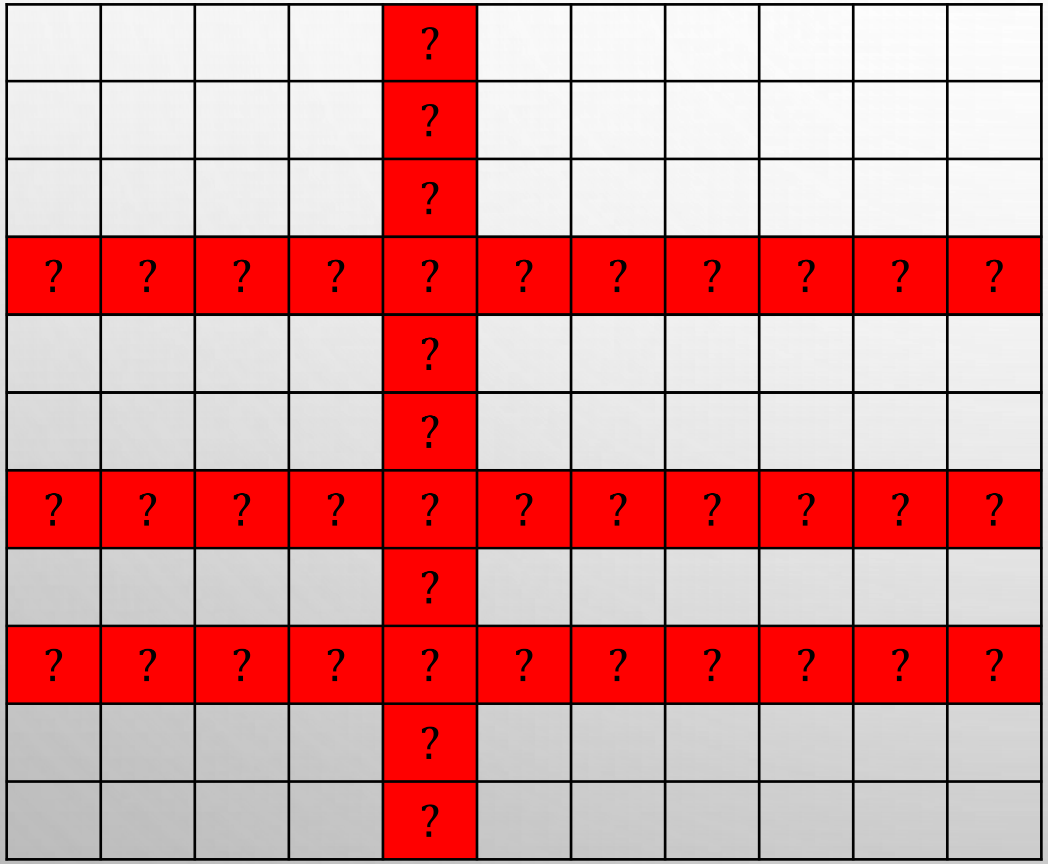
Inf								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
								Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row
$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	$Col.$	Row	Row	Row

Related Work

- **There is a lot of work on *array codes***
  - **Maximum-rank array codes,** Roth 1993
  - **Codes for graph erasures,** YY 2017
  - **EVENODD codes,** Blaum et al. 1995
  - **d-codes,** Schmidt 2010
  - **Etc.**

Related Work: Maximum-Rank Array Codes

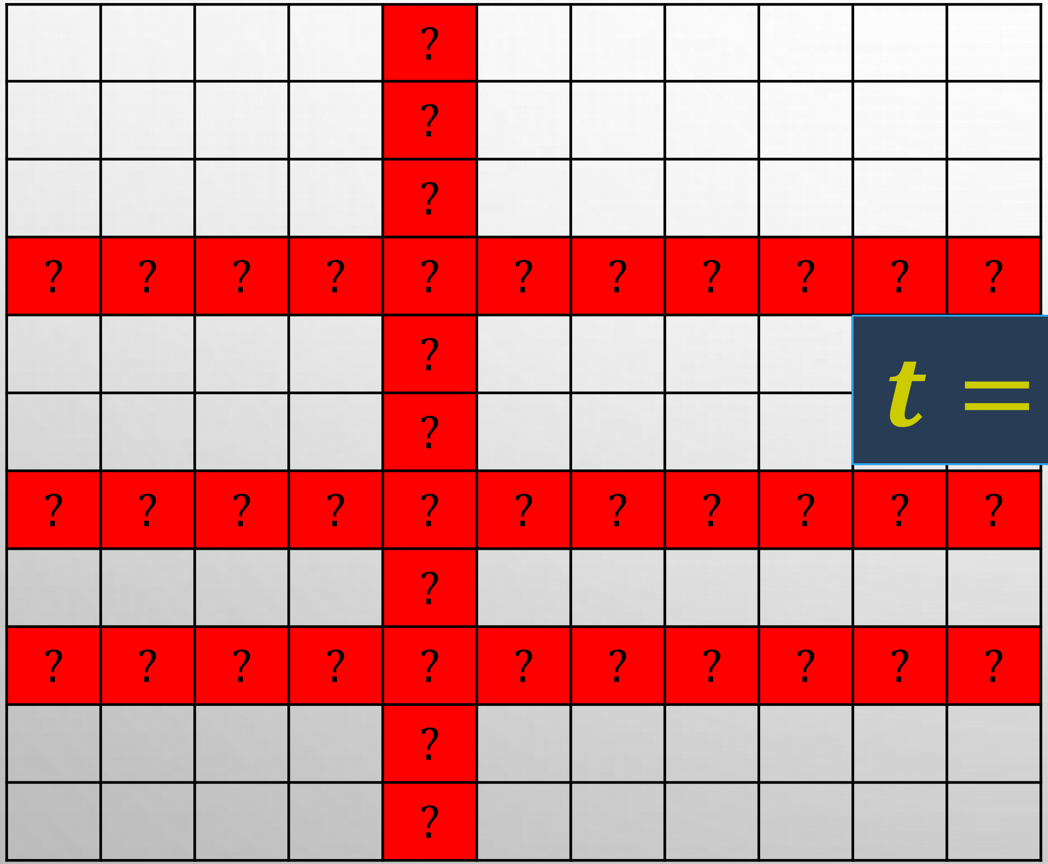
Maximum-rank array codes, Roth 1993



- **Square binary $n \times n$ array codes**
- **Correcting t row/column erasures**
  - *For example* $t = 4$
- **Require $t \cdot n$ redundancy bits**

Related Work: Maximum-Rank Array Codes

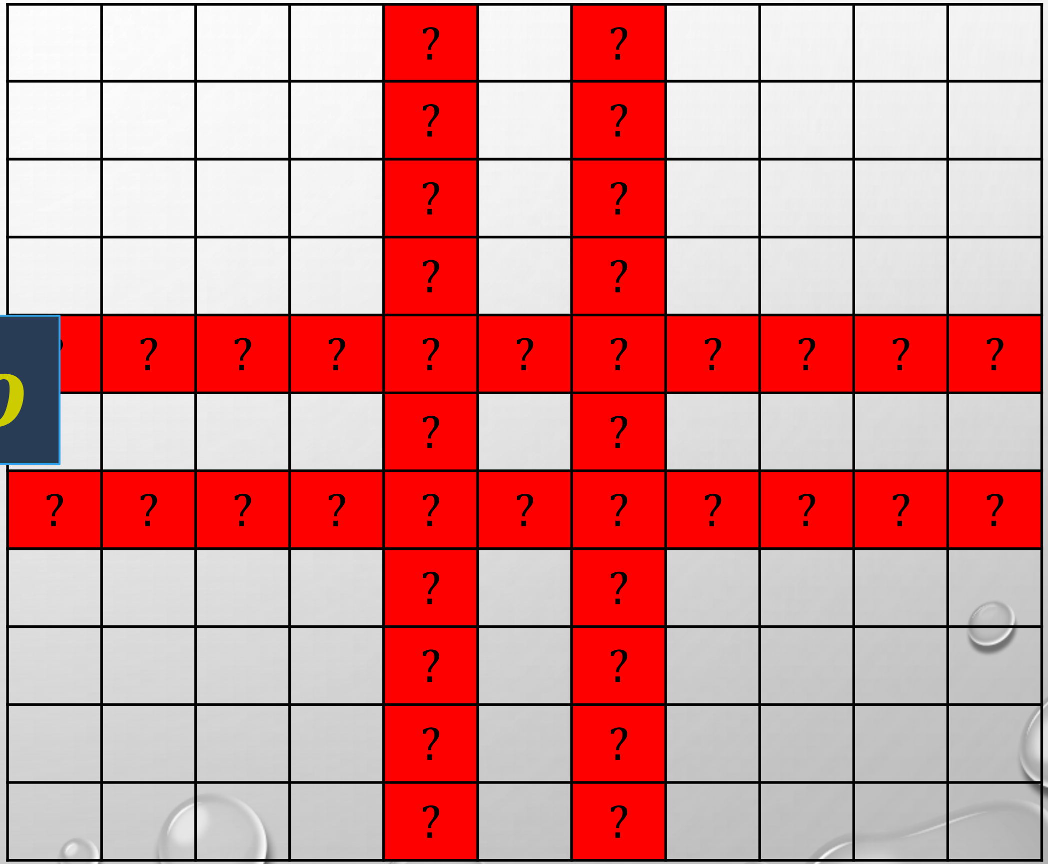
Maximum-rank array codes, Roth 1993



				?						
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						

$$t = 2\rho$$

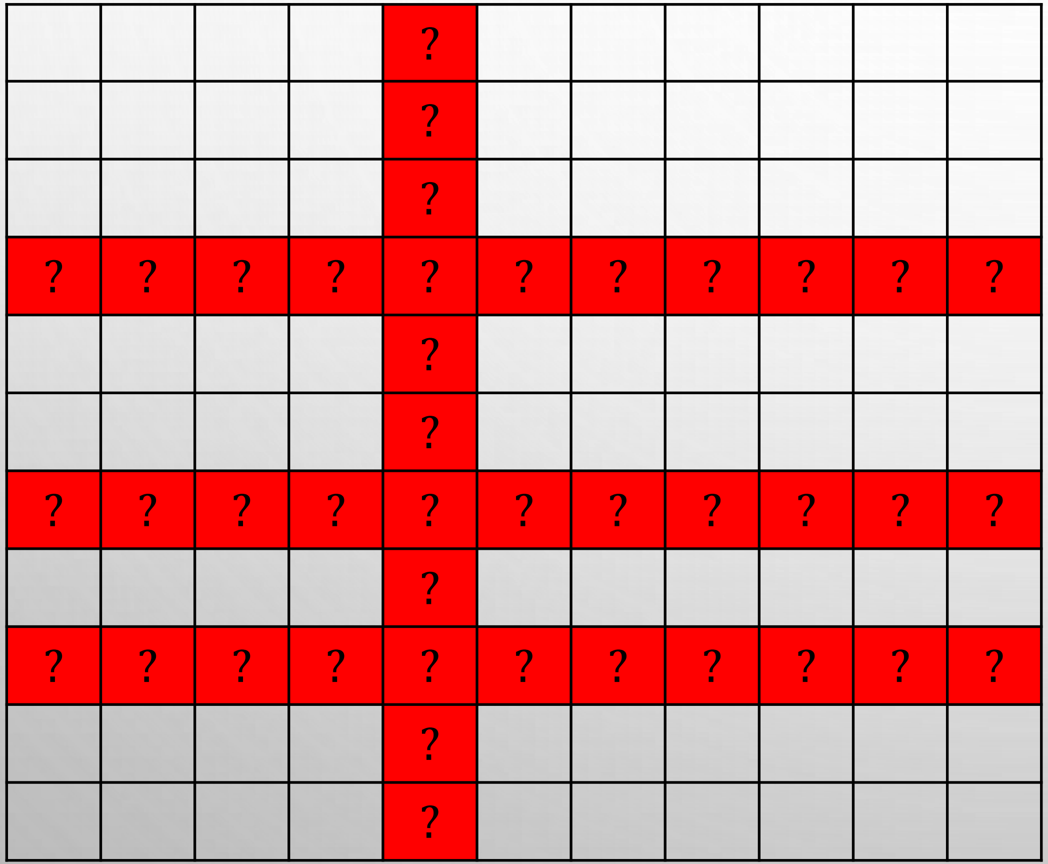
Graph codes



				?		?				
				?		?				
				?		?				
				?		?				
?	?	?	?	?	?	?	?	?	?	?
				?		?				
?	?	?	?	?	?	?	?	?	?	?
				?		?				
				?		?				
				?		?				
				?		?				

Related Work: Maximum-rank array codes

Maximum-rank array codes, Roth 1993



				?						
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						

Codes over a binary field

$$r_G = 2n\rho$$

$$r_G > 2n\rho - \boxed{\rho^2}$$ ← **Penalty**

For $\rho = 2$ only 4 redundancy bits away from optimality

But we have an optimal construction!

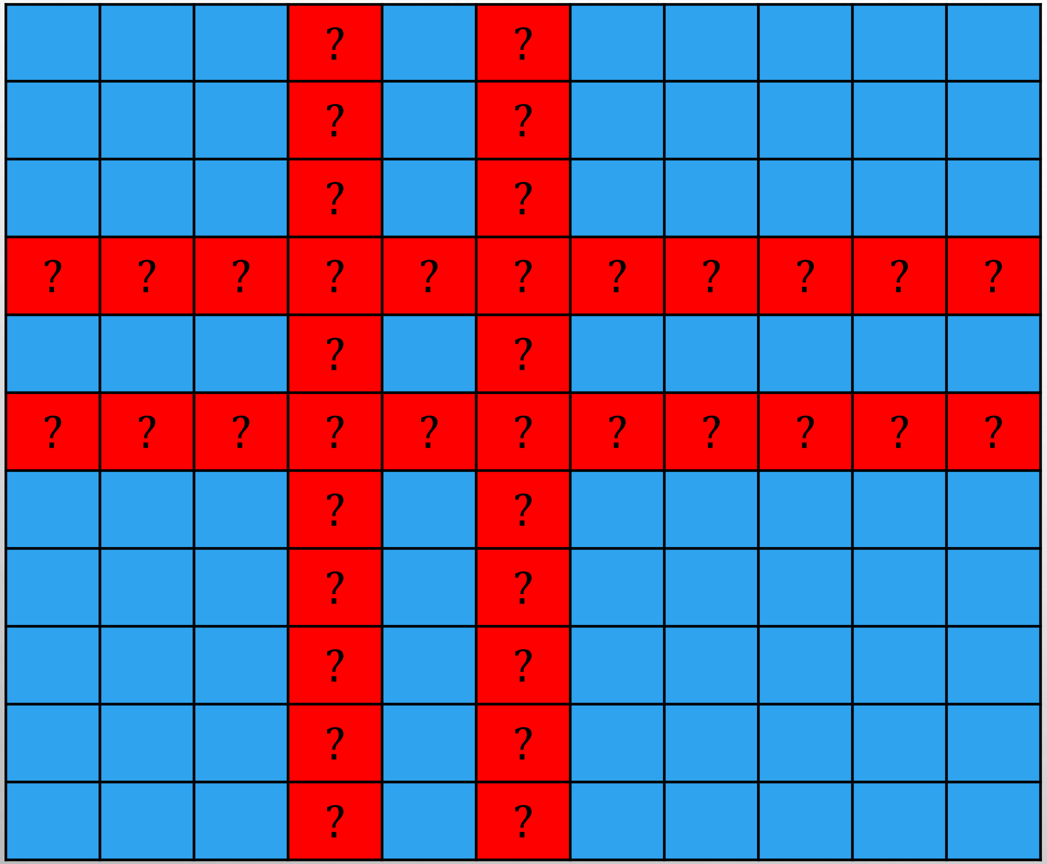
ρ – number of failed nodes

r_G – number of redundancy edges

Optimal Binary Double-Node-Erasure-Correcting Codes $\mathcal{SG}-[n,n-2]$

n **is a prime number**

Codes for Directed Graph Erasures

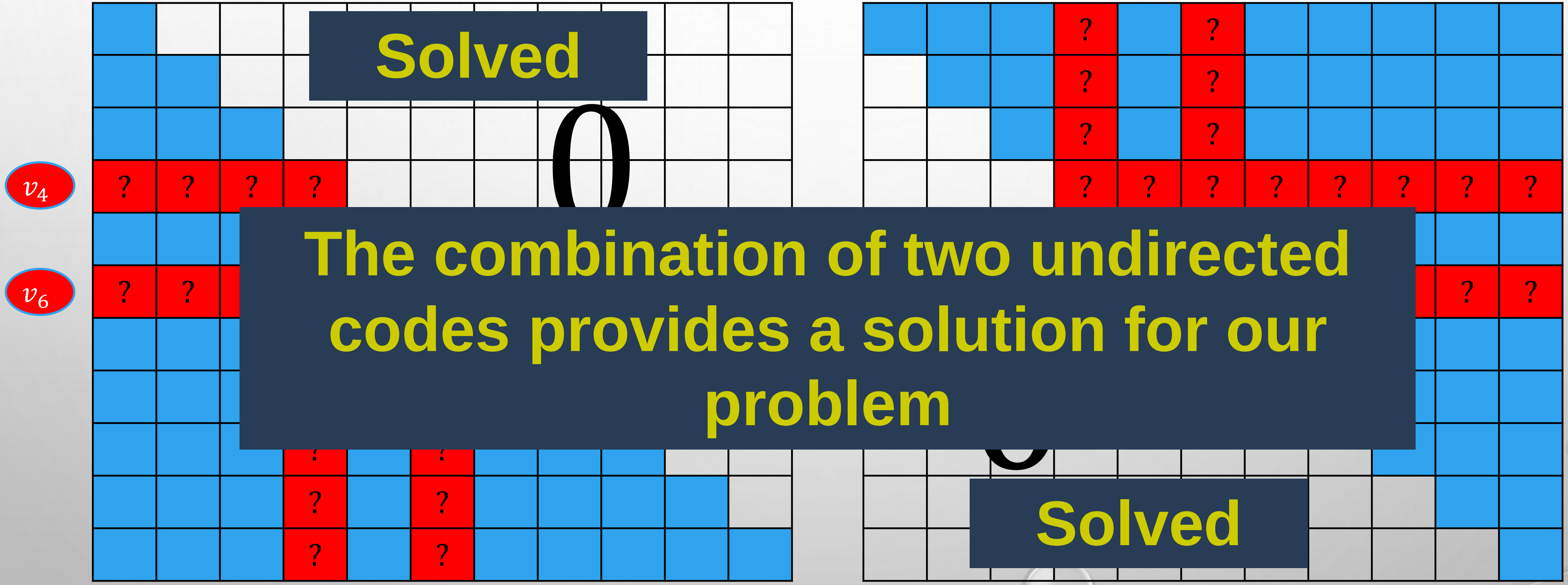


			?		?					
			?		?					
			?		?					
?	?	?	?	?	?	?	?	?	?	?
			?		?					
?	?	?	?	?	?	?	?	?	?	?
			?		?					
			?		?					
			?		?					
			?		?					
			?		?					

Codes for Undirected Graph Erasures

The edges of v_4, v_6 are erased

Undirected pattern of error



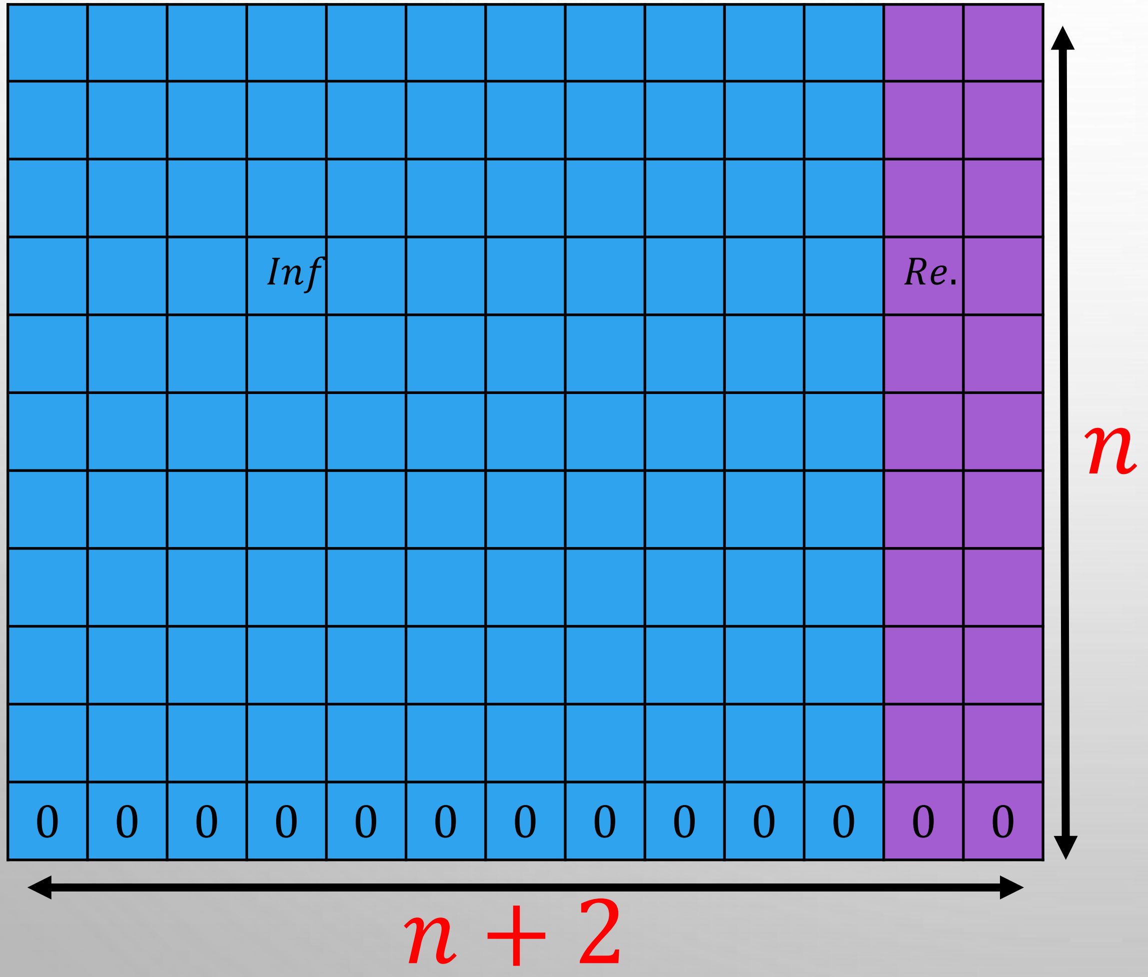
Solved

The combination of two undirected codes provides a solution for our problem

Solved

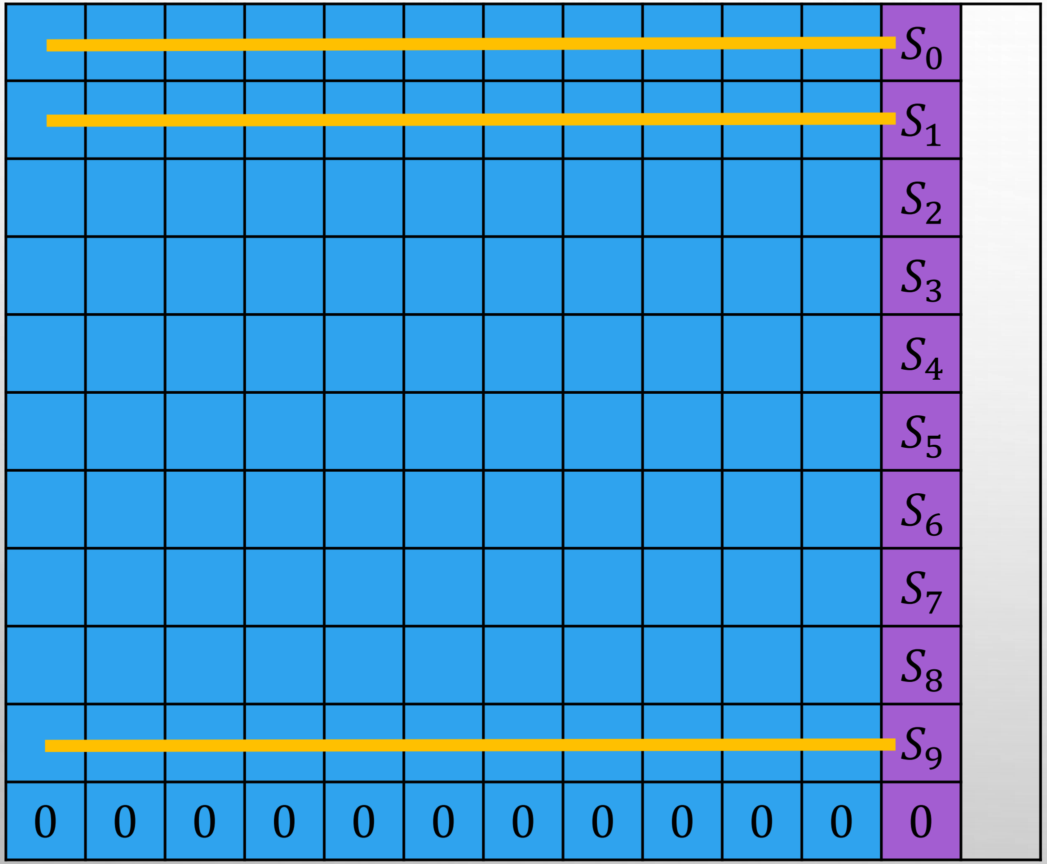
EVENODD Codes

Two columns failure, n is prime

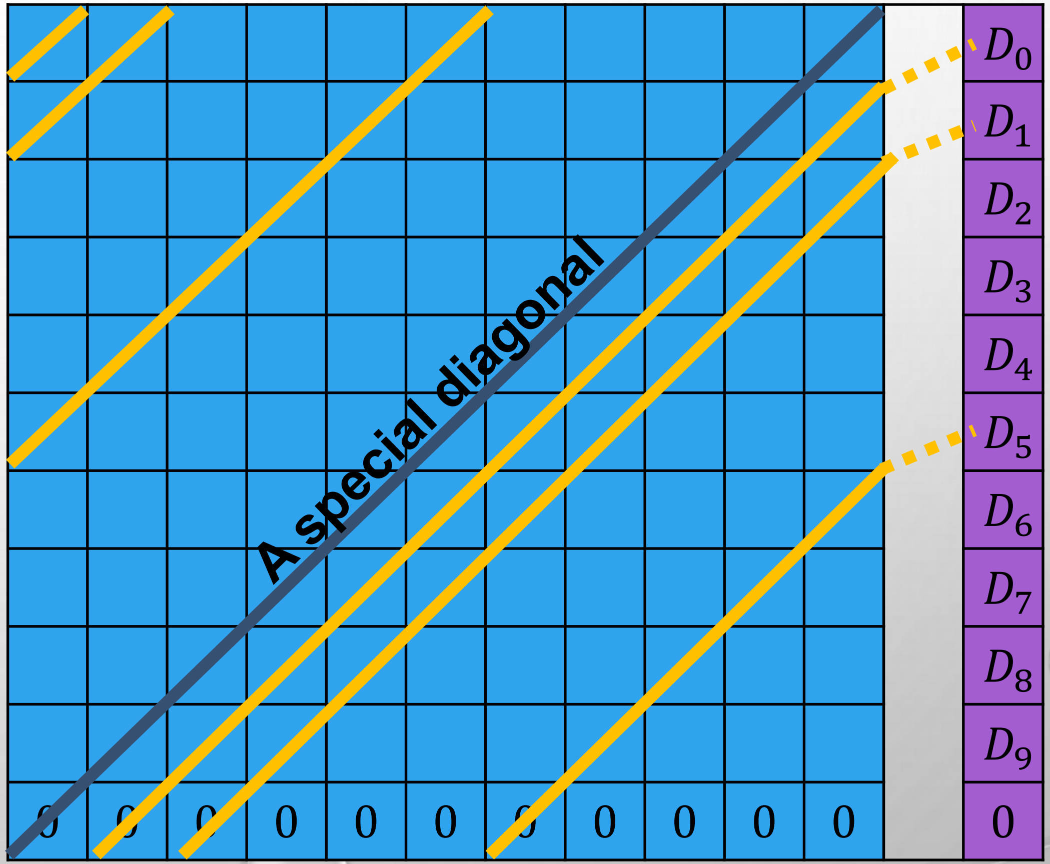


EVENODD Codes

Row parity

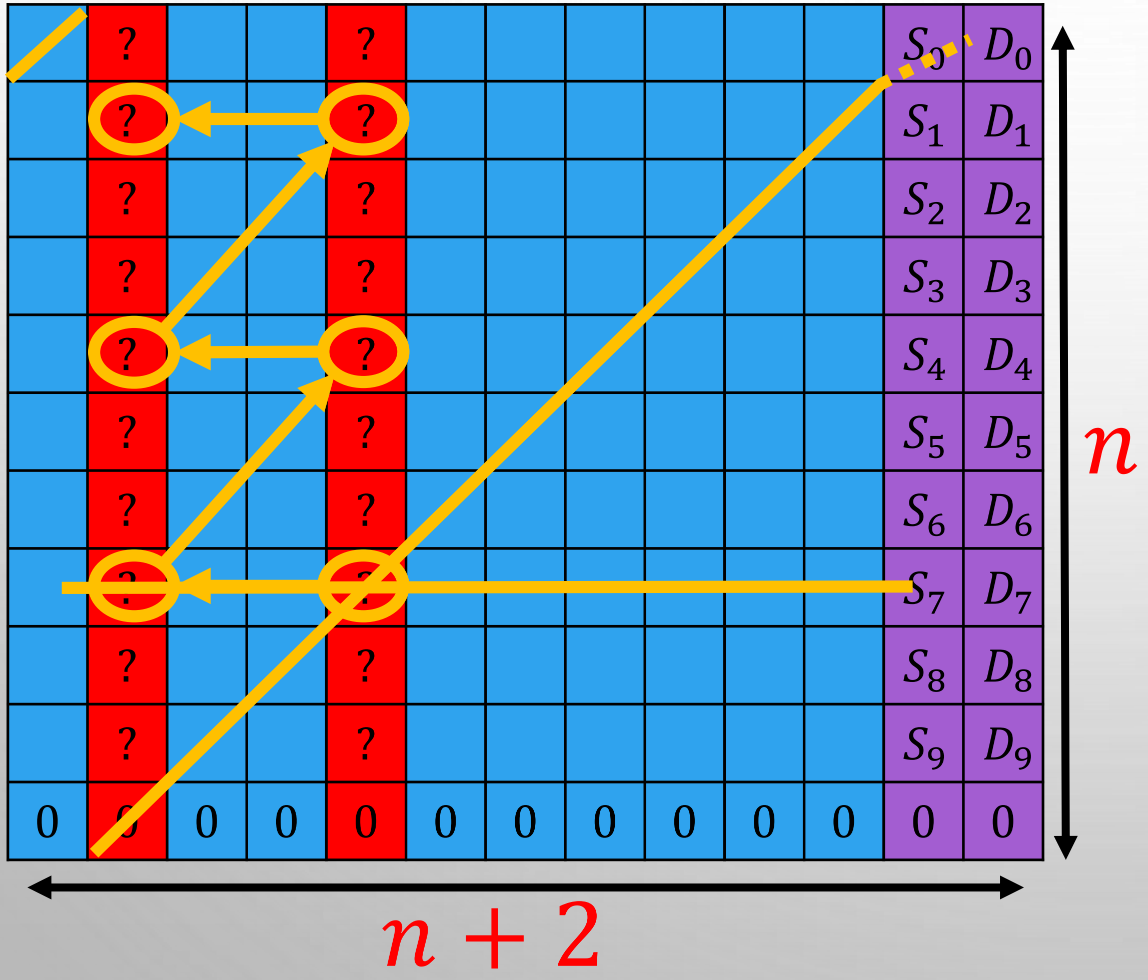


Diagonal parity



EVENODD Codes

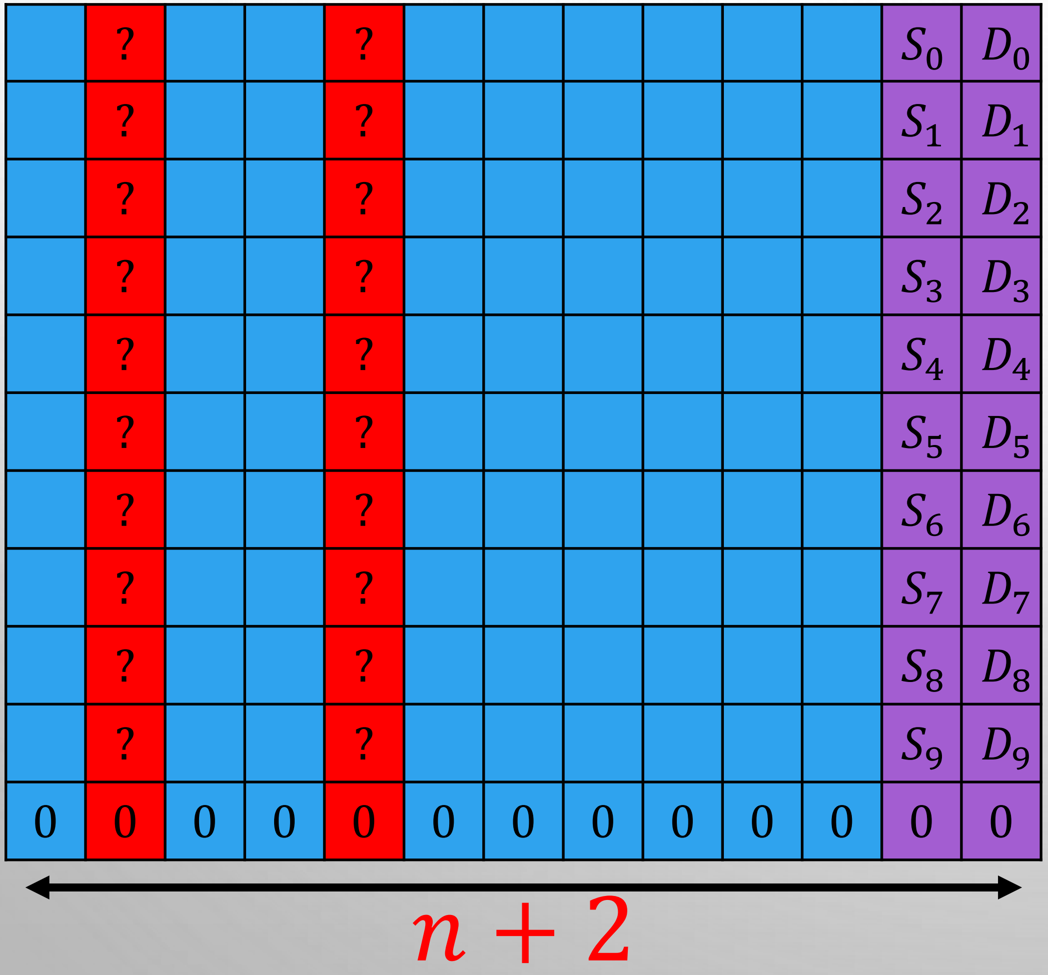
Two columns failure, n is prime



***n* is prime!**
All erasures are corrected!

EVENODD Codes vs Graph Codes

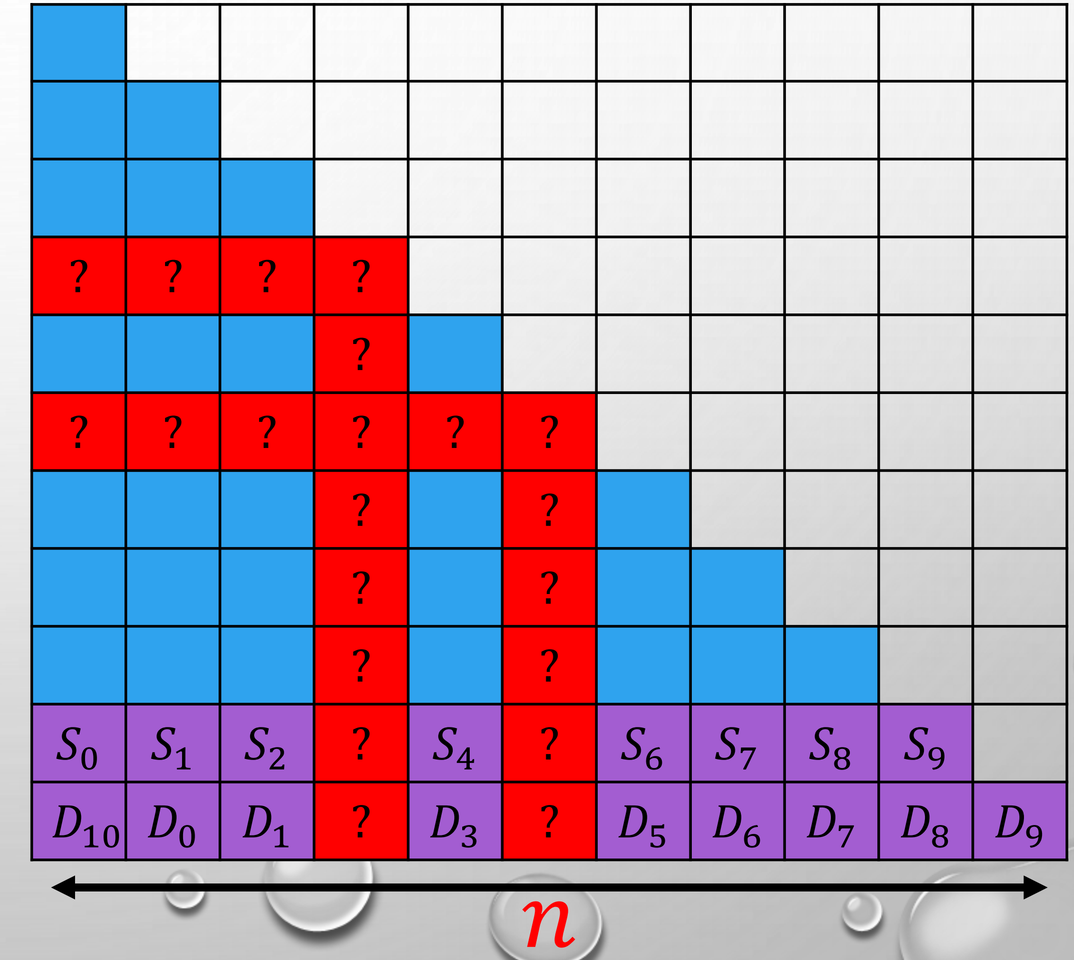
2 column failure, n is prime



🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_0	D_0
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_1	D_1
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_2	D_2
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_3	D_3
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_4	D_4
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_5	D_5
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_6	D_6
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_7	D_7
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_8	D_8
🟦	?	🟦	🟦	?	🟦	🟦	🟦	🟦	🟦	🟦	S_9	D_9
0	0	0	0	0	0	0	0	0	0	0	0	0

$n+2$

2 node failure, n is prime

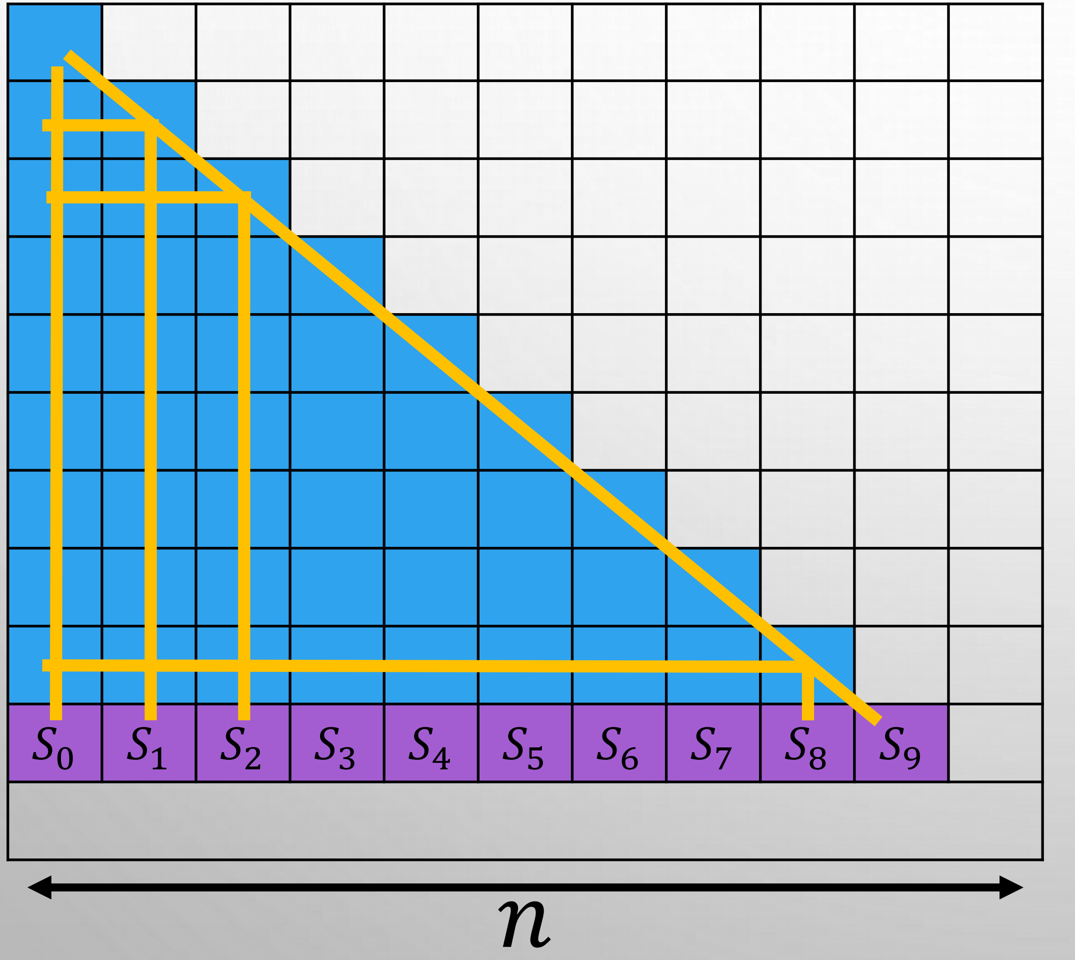


🟦										
🟦	🟦									
🟦	🟦	🟦								
?	?	?	?							
🟦	🟦	🟦	?	🟦						
?	?	?	?	?	?					
🟦	🟦	🟦	?	🟦	?	🟦				
🟦	🟦	🟦	?	🟦	?	🟦	🟦			
🟦	🟦	🟦	?	🟦	?	🟦	🟦	🟦		
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	S_9	
D_{10}	D_0	D_1	?	D_3	?	D_5	D_6	D_7	D_8	D_9

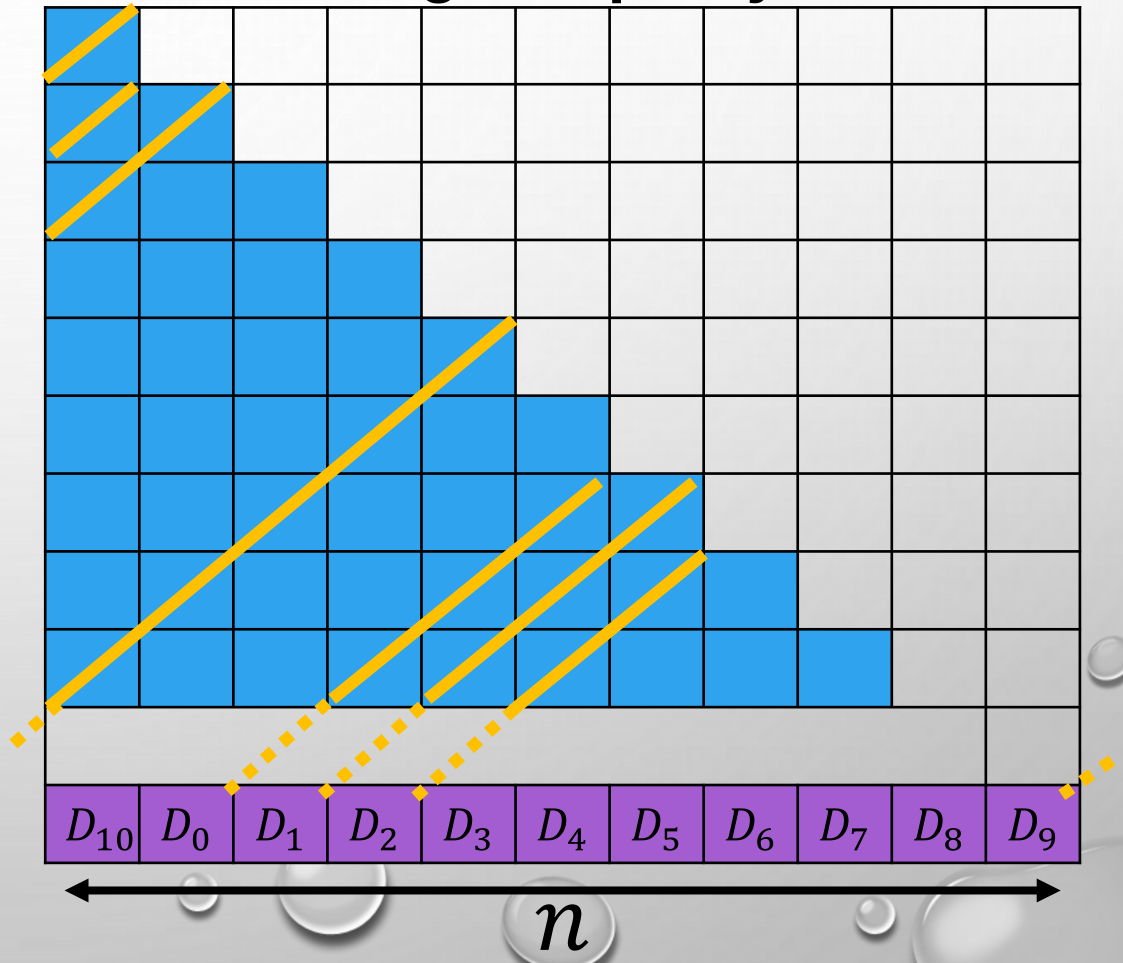
n

EVENODD Codes vs Graph Codes

Neighborhood parity

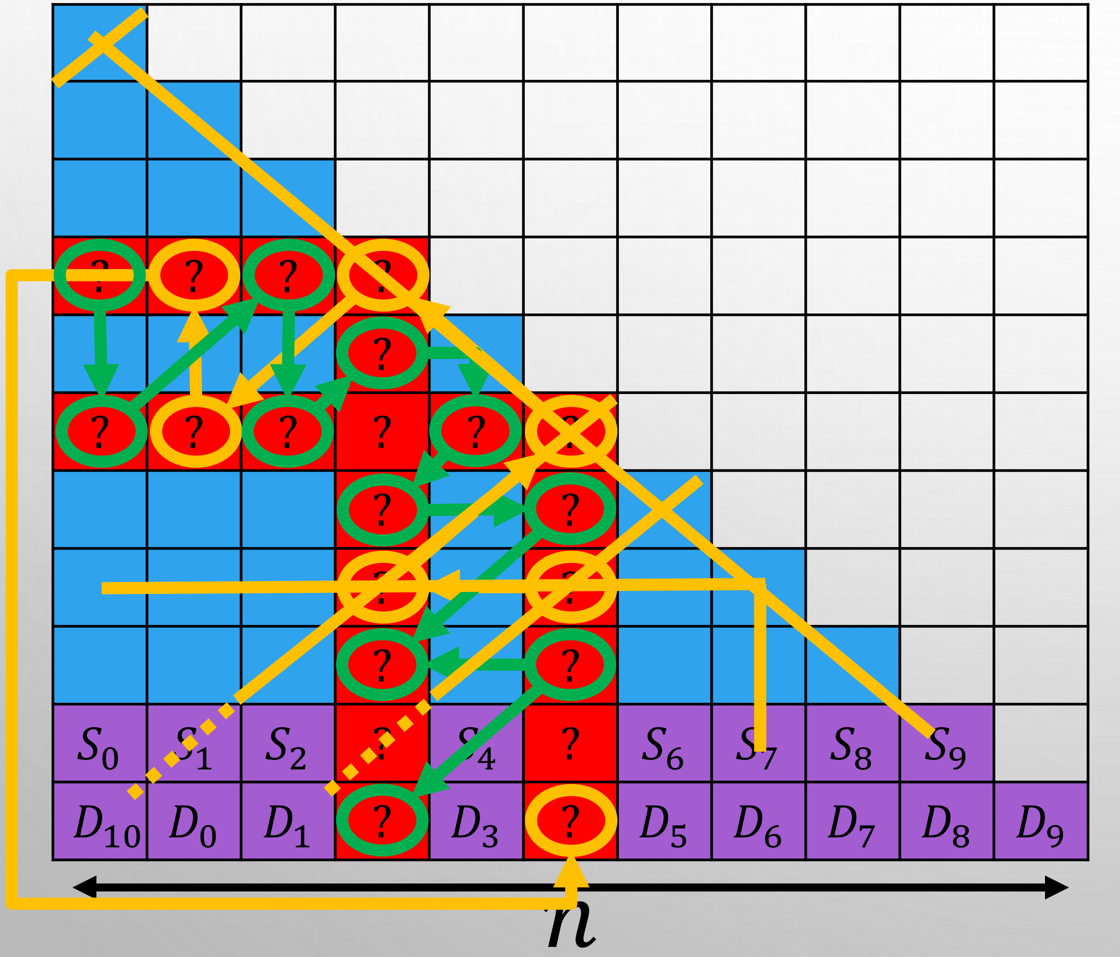


Diagonal parity



EVODD Codes vs Graph Codes

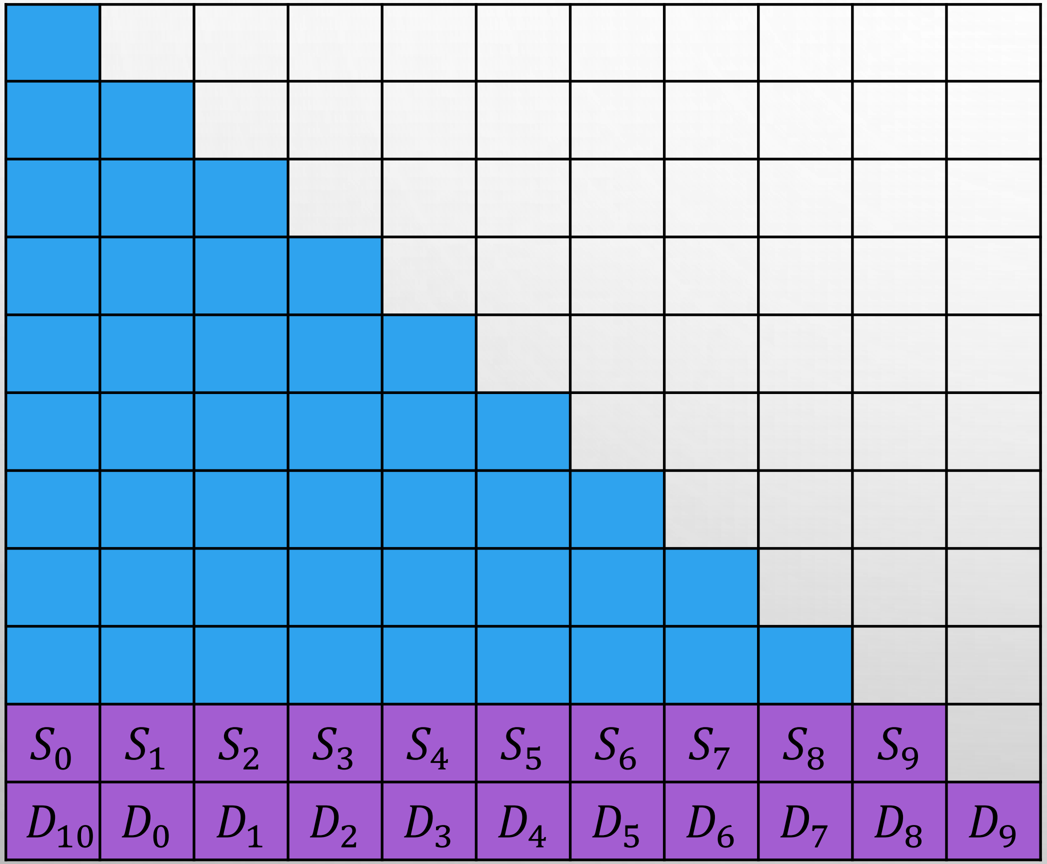
2 node failure, n is prime



- Yellow Loop
- Green Loop

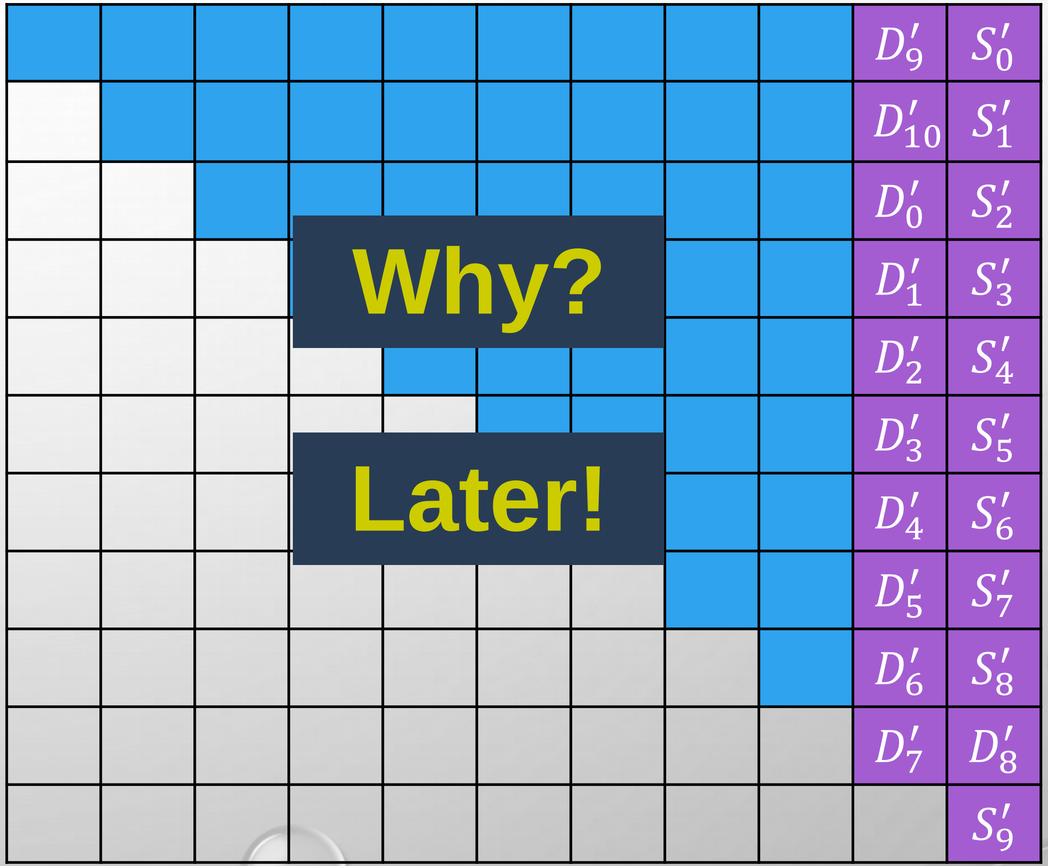
Codes for Undirected Graph Erasures

Switch the roles



🟦										
🟦	🟦									
🟦	🟦	🟦								
🟦	🟦	🟦	🟦							
🟦	🟦	🟦	🟦	🟦						
🟦	🟦	🟦	🟦	🟦	🟦					
🟦	🟦	🟦	🟦	🟦	🟦	🟦				
🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦			
🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦		
S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	
D_{10}	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9

$n-1$ 1st-Neighbourhhod 2nd-Diagonal n



🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	D'_9	S'_0
	🟦	🟦	🟦	🟦	🟦	🟦	🟦	🟦	D'_{10}	S'_1
		🟦	🟦	🟦	🟦	🟦	🟦	🟦	D'_0	S'_2
							🟦	🟦	D'_1	S'_3
				🟦	🟦	🟦	🟦	🟦	D'_2	S'_4
					🟦	🟦	🟦	🟦	D'_3	S'_5
							🟦	🟦	D'_4	S'_6
							🟦	🟦	D'_5	S'_7
								🟦	D'_6	S'_8
									D'_7	D'_8
										S'_9

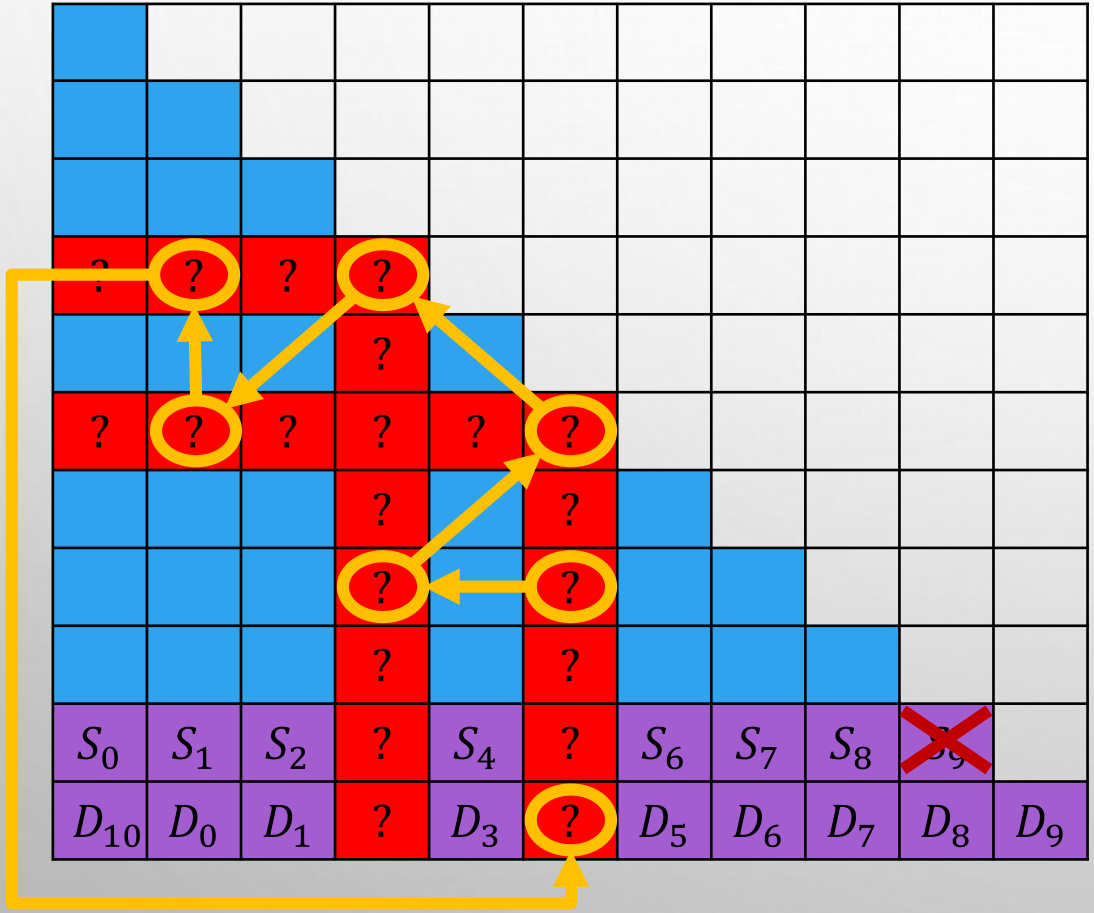
Why?

Later!

$n-1$ 1st-Diagonal 2nd-Neighbourhhod n

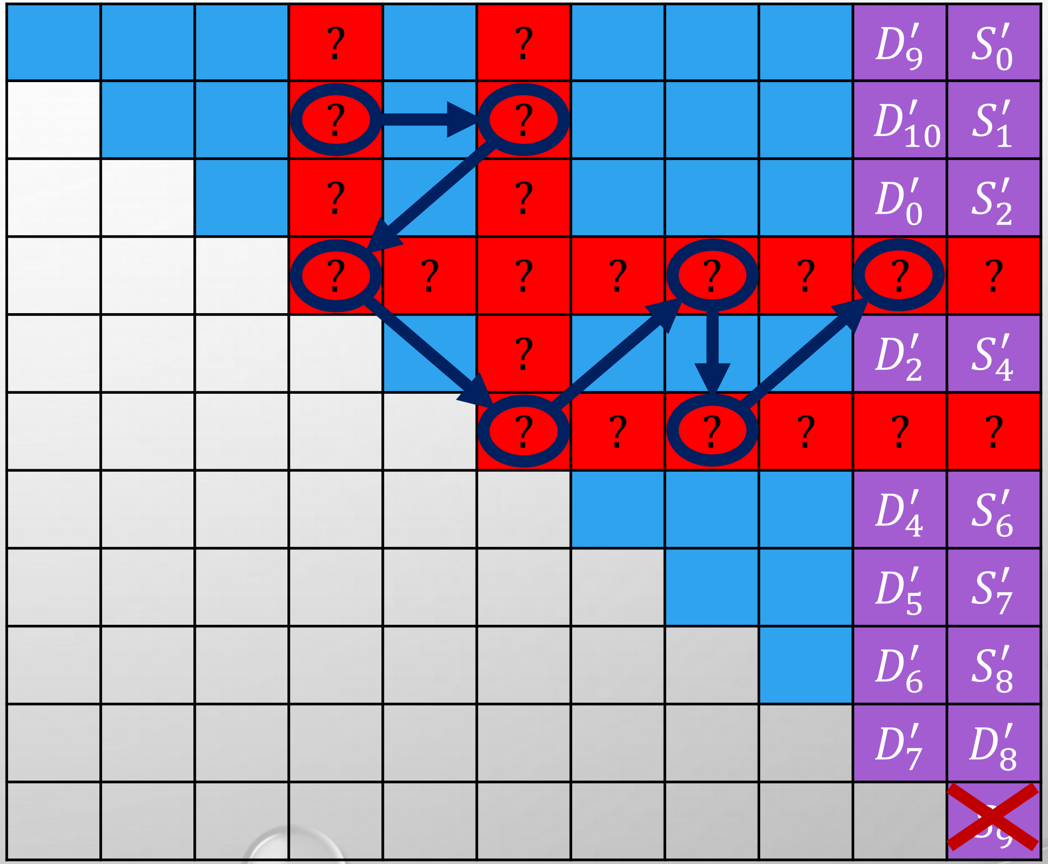
Codes for Directed Graph Erasures

Yellow Loop



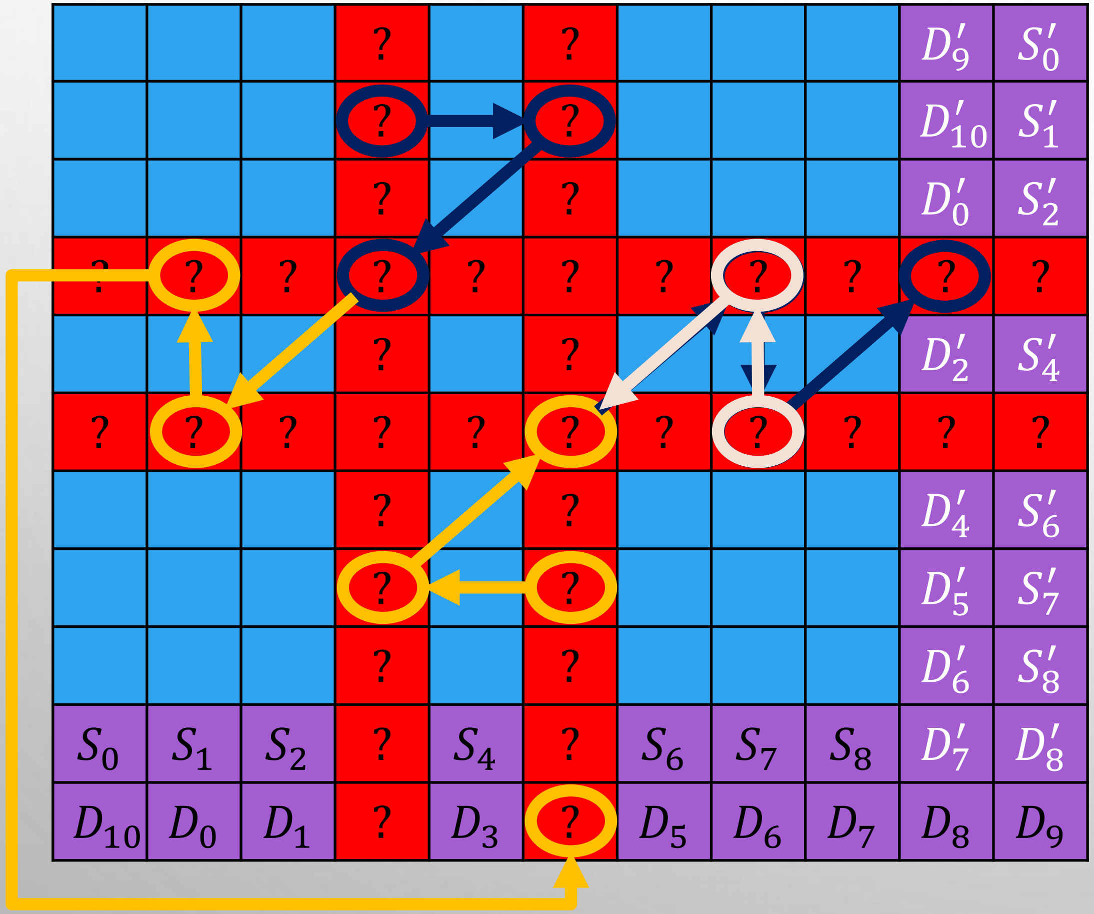
🟦										
🟦	🟦									
🟦	🟦	🟦								
?	?	?	?							
🟦	🟦	🟦	?	🟦						
?	?	?	?	?	?					
🟦	🟦	🟦	?	🟦	?	🟦				
🟦	🟦	🟦	?	🟦	?	🟦	🟦			
🟦	🟦	🟦	?	🟦	?	🟦	🟦	🟦		
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	~~S_9~~	
D_{10}	D_0	D_1	?	D_3	?	D_5	D_6	D_7	D_8	D_9

Dark-Blue Loop



🟦	🟦	🟦	?	🟦	?	🟦	🟦	🟦	D'_9	S'_0
	🟦	🟦	?	🟦	?	🟦	🟦	🟦	D'_{10}	S'_1
		🟦	?	🟦	?	🟦	🟦	🟦	D'_0	S'_2
			?	?	?	?	?	?	?	?
				🟦	?	🟦	🟦	🟦	D'_2	S'_4
					?	?	?	?	?	?
						🟦	🟦	🟦	D'_4	S'_6
							🟦	🟦	D'_5	S'_7
								🟦	D'_6	S'_8
									D'_7	D'_8
										~~S_9~~

Codes for Directed Graph Erasures



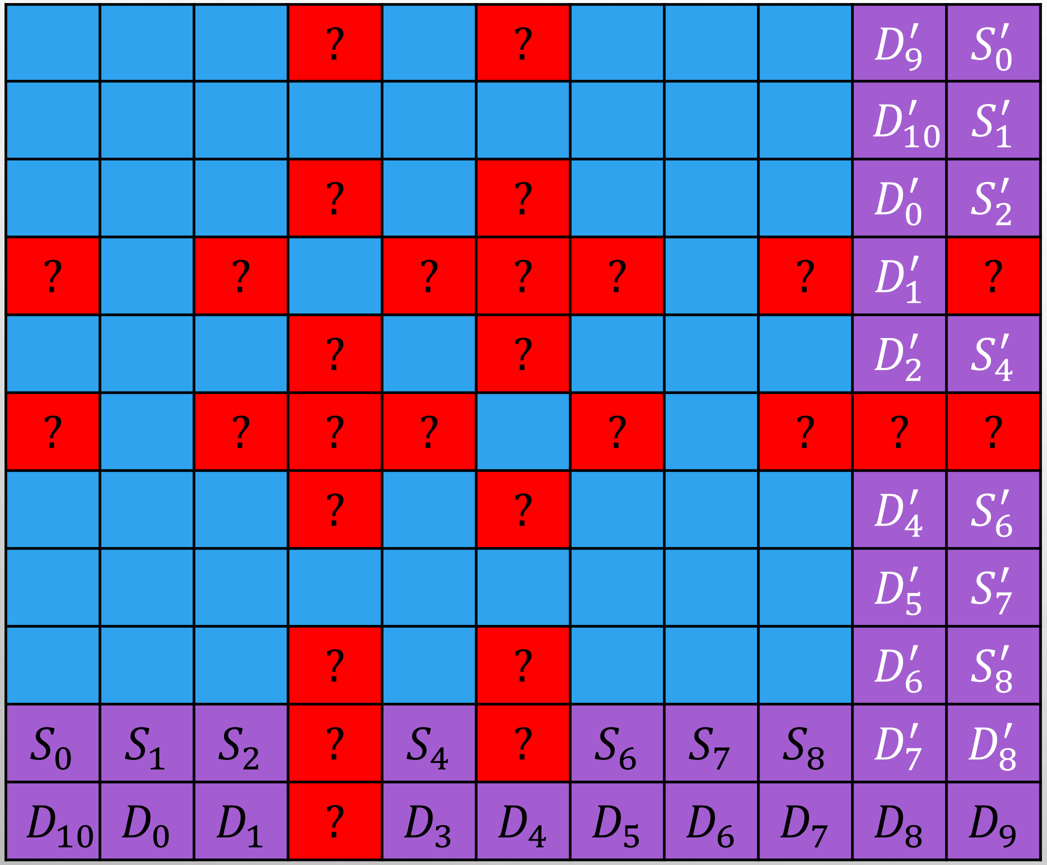
			?		?				D'_9	S'_0
			?		?				D'_{10}	S'_1
			?		?				D'_0	S'_2
?	?	?	?	?	?	?	?	?	?	?
			?		?				D'_2	S'_4
?	?	?	?	?	?	?	?	?	?	?
			?		?				D'_4	S'_6
			?		?				D'_5	S'_7
			?		?				D'_6	S'_8
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	?	D_3	?	D_5	D_6	D_7	D_8	D_9

Does switching the roles help?

YES!!

The pink loop is a mirror of the yellow loop

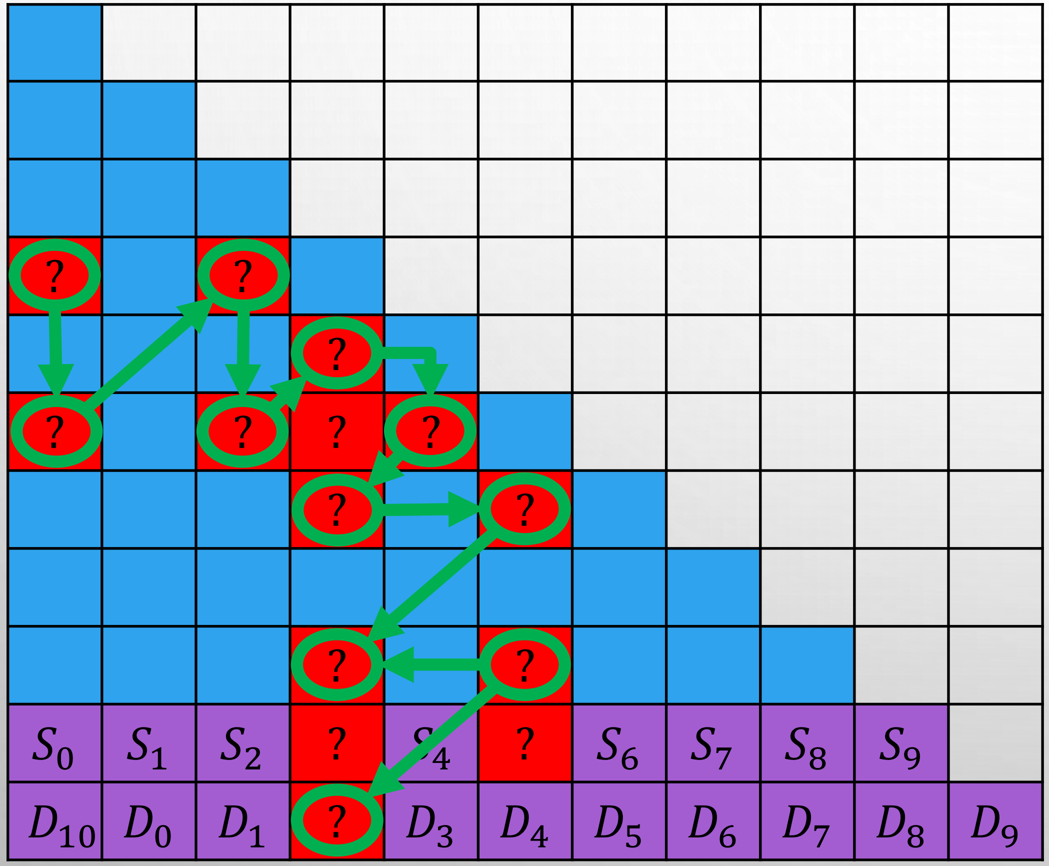
Codes for Directed Graph Erasures



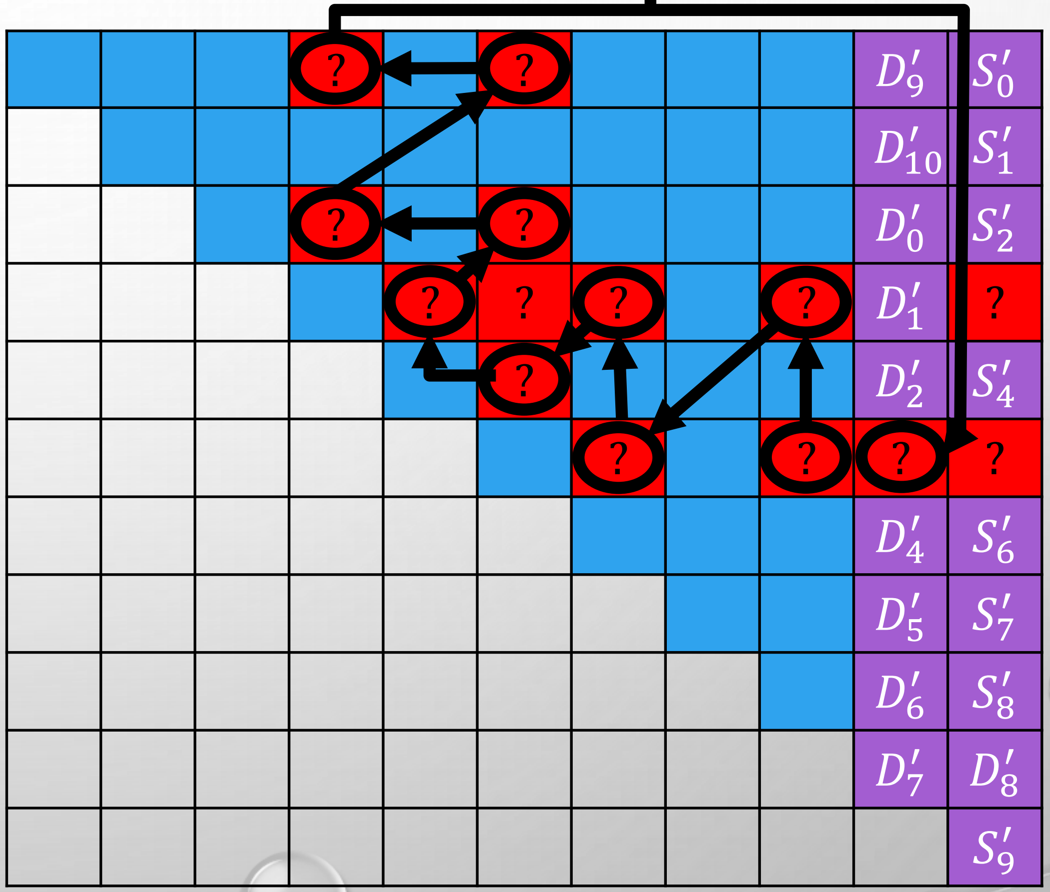
			?		?				D'_9	S'_0
									D'_{10}	S'_1
			?		?				D'_0	S'_2
?		?		?	?	?		?	D'_1	?
			?		?				D'_2	S'_4
?		?	?	?		?		?	?	?
			?		?				D'_4	S'_6
									D'_5	S'_7
			?		?				D'_6	S'_8
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	?	D_3	D_4	D_5	D_6	D_7	D_8	D_9

Codes for Directed Graph Erasures

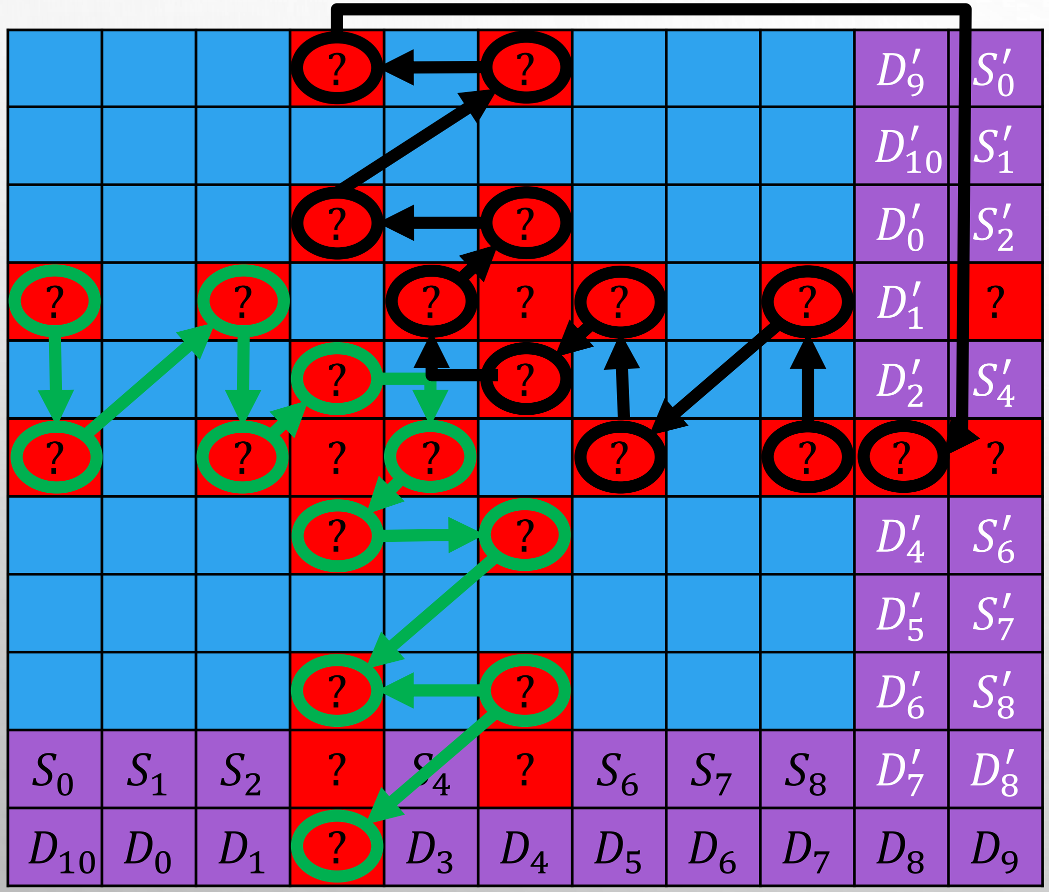
Green Loop



Black Loop



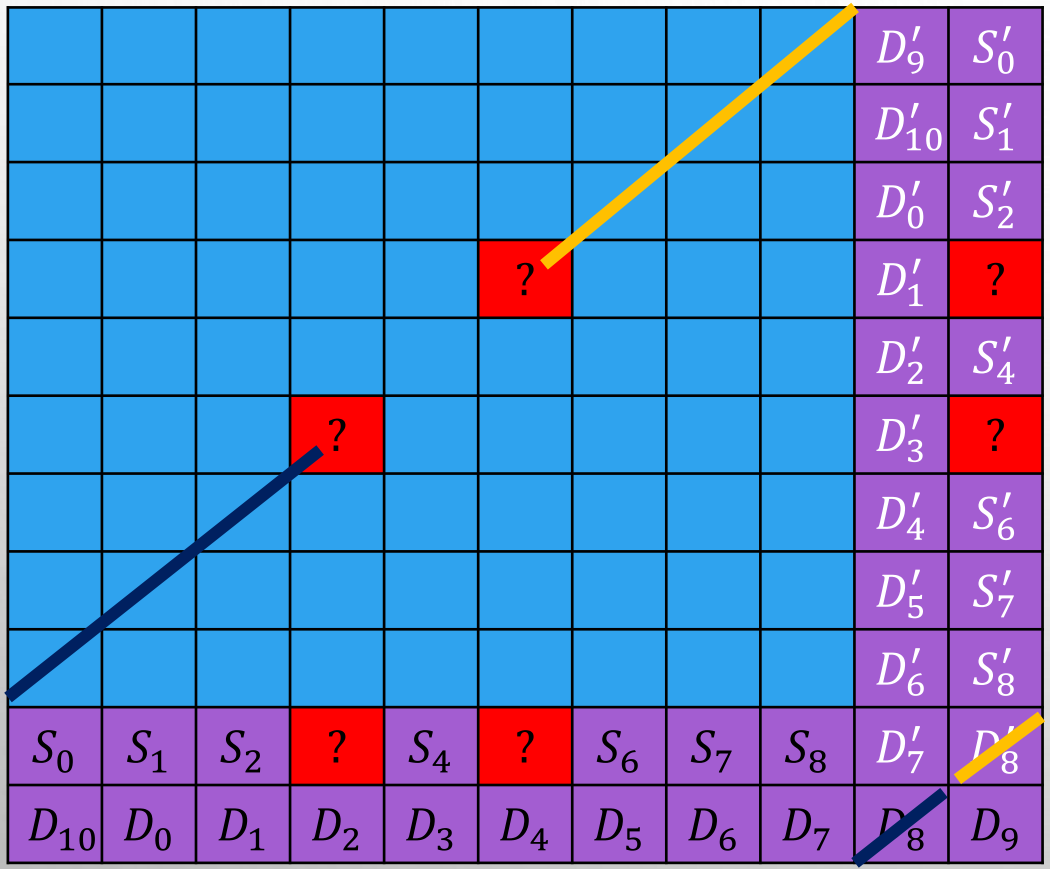
Codes for Directed Graph Erasures



			?		?				D'_9	S'_0
									D'_{10}	S'_1
			?		?				D'_0	S'_2
?		?		?	?	?		?	D'_1	?
			?		?				D'_2	S'_4
?		?	?	?		?		?	?	?
			?		?				D'_4	S'_6
									D'_5	S'_7
			?		?				D'_6	S'_8
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	?	D_3	D_4	D_5	D_6	D_7	D_8	D_9

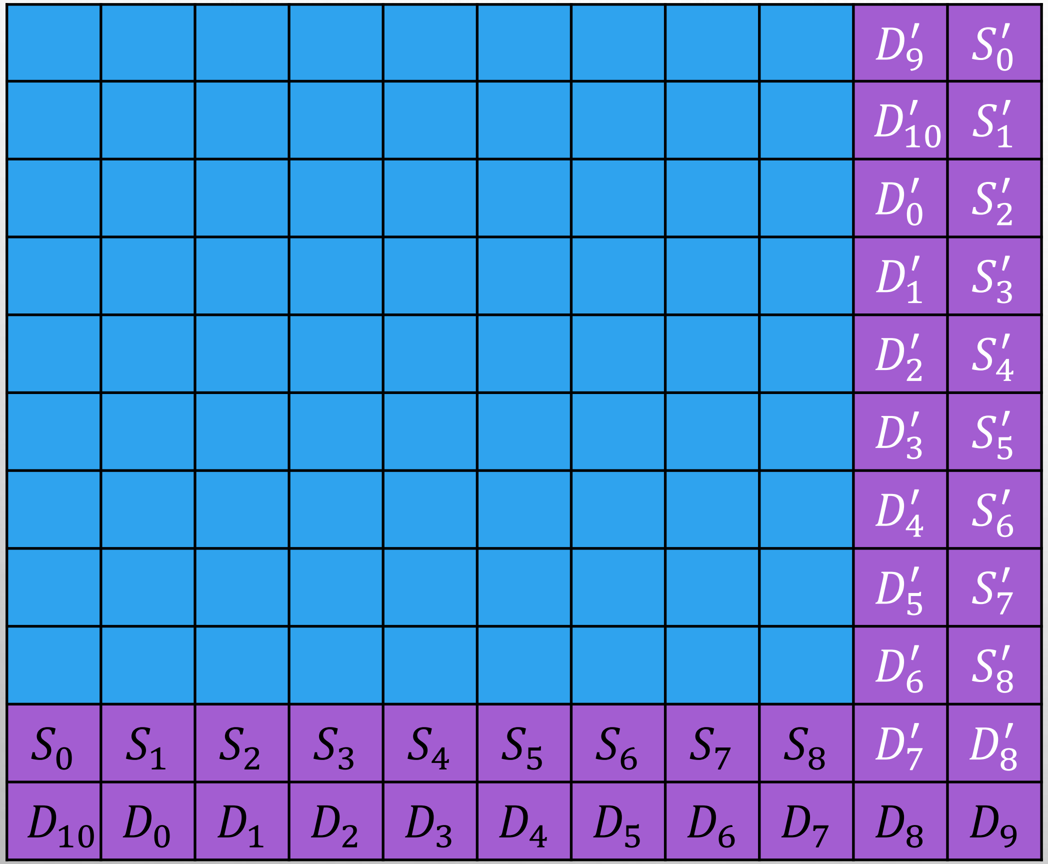
Main diagonal parity isn’t required!

Codes for Directed Graph Erasures



									D'_9	S'_0
									D'_{10}	S'_1
									D'_0	S'_2
					?				D'_1	?
									D'_2	S'_4
			?						D'_3	?
									D'_4	S'_6
									D'_5	S'_7
									D'_6	S'_8
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9

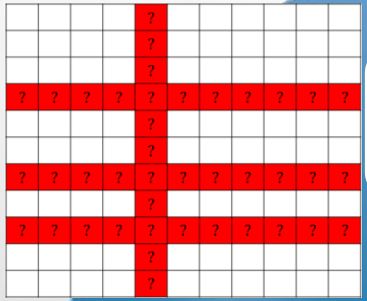
Codes for Directed Graph Erasures



									D'_9	S'_0
									D'_{10}	S'_1
									D'_0	S'_2
									D'_1	S'_3
									D'_2	S'_4
									D'_3	S'_5
									D'_4	S'_6
									D'_5	S'_7
									D'_6	S'_8
S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9

Codes over Directed Graphs - Summary

The world of our codes over a binary field

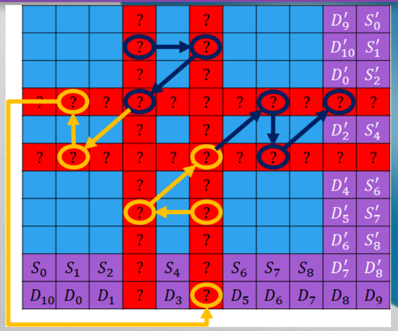


				?						
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
?	?	?	?	?	?	?	?	?	?	?
				?						
				?						

$\mathcal{G}$ $-[n, k_G]$
$r_G = 2n\rho$

Roth 1993

Our main result



			?		?				D'_9	S'_0
			?		?				D'_{10}	S'_1
			?		?				D'_0	S'_2
?	?	?	?	?	?	?	?	?	?	?
			?		?				D'_2	S'_4
?	?	?	?	?	?	?	?	?	?	?
			?		?				D'_4	S'_6
			?		?				D'_5	S'_7
			?		?				D'_6	S'_8
S_0	S_1	S_2	?	S_4	?	S_6	S_7	S_8	D'_7	D'_8
D_{10}	D_0	D_1	?	D_3	?	D_5	D_6	D_7	D_8	D_9

$\mathcal{SG} - [n$ **prime**$, n-2]$
$\rho = 2$

➡

$\mathcal{SG} - [n = 4{,}6{,}8\ , n-2]$
$\rho = 2$

Exhaustive search

$\mathcal{SG} - [n, 2]$
$\rho = n-2$

Isn't shown

Open problem

Thank You